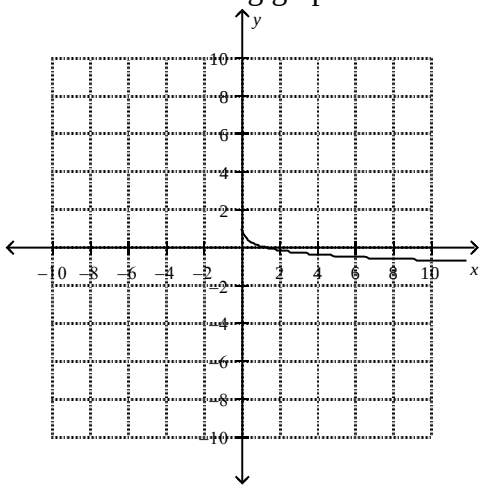
Math 3201 Test

Multiple Choice
Identify the choice that best completes the statement or answers the question.

____ **1.** Match the following graph with its function.



A. $y = -\frac{1}{3} \ln x$
B. $y = 3 \log x$
C. $y = -\frac{1}{3}(3)^x$
D. $y = 0.3(10)^x$

____ **2.** Which exponential equation correctly represents the logarithmic equation $y = \log 50$?

A. $50^y = 10$
B. $10^y = 50$
C. $y^{50} = 10$
D. $y^{10} = 50$

____ **3.** Which logarithmic equation correctly represents the exponential equation $10^7 = x$?

A. $x = \log 7$
B. $x = \log 10$
C. $7 = \log x$
D. $10 = \log x$

____ **4.** Calculate the pH of a solution with a hydrogen ion concentration of 6.5×10^{-6} mol/L.
Recall that pH, $p(x)$, is defined by the equation
$p(x) = -\log x$
where the concentration of hydrogen ions, x, in a solution is measured in moles per litre.

A. 6.5
B. –5.2
C. –6.5
D. 5.2

____ **5.** Which expression is equivalent to $\ln\left(\frac{8}{5}\right)$?

A. ln 8 – ln 5
B. ln 5 – ln 8
C. 8 ln 5
D. ln 0.625

____ **6.** Evaluate:
$\log_{12} 16 + 2\log_{12} 3$

A. –4
B. 4
C. 2
D. 0

____ **7.** Evaluate:
$3\log_8 24 - 3\log_8 3$

A. 3
B. 7
C. 1
D. 64

____ **8.** Which logarithmic expression is **not** equivalent to the others?

A. $\dfrac{\log 6}{\log\left(\frac{1}{4}\right)}$

B. $\log_{10} 1.5$

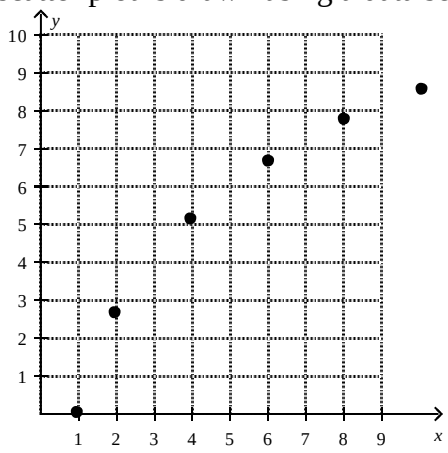
C. $\dfrac{\log 6}{2\log 0.5}$

D.

____ **9.** The equation of the logarithmic function that models a data set is $y = 43.9 - 8.7 \ln x$. Extrapolate the value of y when $x = 120$.

A. $y = 1.2$
B. $y = 2.2$
C. $y = 3.2$
D. $y = 4.2$

____ **10.** A scatter plot is drawn using a data set. Identify the equation of the curve of best fit.



A. $y = 8.5 + \log x$
B. $y = 8.5 + \ln x$
C. $y = 8.5 \log x$
D. $y = 8.5 \ln x$

Short Answer: Answer the following in the space provided.

1. Solve each equation. Round your answers to three decimal places.

a) $5^{x+3} = 450$

b) $4^{x+3} = 6^{x-1}$

2. Evaluate the following using the laws of logarithms:

a. $3\log_6(2) + \log_6(27)$

b. $\log_5(2.5) + 2\log_5(10) - \log_5(2)$

3. Evaluate: $\log_7 100$

4. $1600 is invested at 3% per year, compounded monthly. In which year after the initial investment, will the investment reach $2000? Use the compound interest formula:
$A = P(1+i)^n$
Show your work.

5. A laboratory that uses radioactive substances received a shipment of 122 g of bismuth-210. Only 6.64 g of the bismuth-210 remained 21.0 days later. Determine the half-life of bismuth-210 algebraically using logarithms, to the nearest tenth of a day. The half-life equation is
$A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$
where A represents the amount of the substance remaining, A_0 represents the initial amount of the substance, t represents the time, and h represents the time at which only half of the substance remains.

Math 3201 Test
Answer Section

MULTIPLE CHOICE

1. ANS: A PTS: 1 DIF: Grade 12 REF: Lesson 7.1
OBJ: 4.1 Express a logarithmic equation as an exponential equation and vice versa. | 6.1 Describe, orally and in written form, the characteristics of an exponential or logarithmic function by analyzing its graph. | 6.2 Describe, orally and in written form, the characteristics of an exponential or logarithmic function by analyzing its their equation. | 6.3 Match equations in a given set to their corresponding graphs.
TOP: Characteristics of logarithmic functions with base 10 and base e
KEY: logarithmic function

2. ANS: B PTS: 1 DIF: Grade 12 REF: Lesson 7.1
OBJ: 4.1 Express a logarithmic equation as an exponential equation and vice versa. | 6.1 Describe, orally and in written form, the characteristics of an exponential or logarithmic function by analyzing its graph. | 6.2 Describe, orally and in written form, the characteristics of an exponential or logarithmic function by analyzing its their equation. | 6.3 Match equations in a given set to their corresponding graphs.
TOP: Characteristics of logarithmic functions with base 10 and base e
KEY: logarithmic function | exponential function

3. ANS: C PTS: 1 DIF: Grade 12 REF: Lesson 7.1
OBJ: 4.1 Express a logarithmic equation as an exponential equation and vice versa. | 6.1 Describe, orally and in written form, the characteristics of an exponential or logarithmic function by analyzing its graph. | 6.2 Describe, orally and in written form, the characteristics of an exponential or logarithmic function by analyzing its their equation. | 6.3 Match equations in a given set to their corresponding graphs.
TOP: Characteristics of logarithmic functions with base 10 and base e
KEY: logarithmic function | exponential function

4. ANS: D PTS: 1 DIF: Grade 12 REF: Lesson 7.2
OBJ: 4.1 Express a logarithmic equation as an exponential equation and vice versa. | 4.2 Determine the value of a logarithmic expression, such as log_2 8, without technology. | 4.5 Determine the approximate value of a logarithmic expression, such as log_2 9, with technology.
TOP: Evaluating logarithmic expressions KEY: logarithmic function

5. ANS: A PTS: 1 DIF: Grade 12 REF: Lesson 7.3
OBJ: 4.3 Develop the laws of logarithms, using numeric examples and the exponent laws. | 4.4 Determine an equivalent expression for a logarithmic expression by applying the Solving exponential equations using logarithms. TOP: Laws of logarithms
KEY: logarithmic function

6. ANS: C PTS: 1 DIF: Grade 12 REF: Lesson 7.3
OBJ: 4.3 Develop the laws of logarithms, using numeric examples and the exponent laws. | 4.4 Determine an equivalent expression for a logarithmic expression by applying the Solving exponential equations using logarithms. TOP: Laws of logarithms
KEY: logarithmic function

7. ANS: A PTS: 1 DIF: Grade 12 REF: Lesson 7.3
OBJ: 4.3 Develop the laws of logarithms, using numeric examples and the exponent laws. | 4.4 Determine an equivalent expression for a logarithmic expression by applying the Solving exponential equations using logarithms. TOP: Laws of logarithms
KEY: logarithmic function

8. ANS: B PTS: 1 DIF: Grade 12 REF: Lesson 7.4
OBJ: 5.2 Determine the solution of an exponential equation in which the bases are not powers of one another. | 5.3 Solve problems that involve the application of exponential equations to loans, mortgages and investments.
TOP: Solving exponential equations using logarithms KEY: logarithmic function

9. ANS: B PTS: 1 DIF: Grade 12 REF: Lesson 7.5
OBJ: 6.4 Graph data and determine the exponential or logarithmic function that best approximates the data. | 6.5 Interpret the graph of an exponential or logarithmic function that models a situation, and explain the reasoning. | 6.6 Solve, using technology, a contextual problem that involves data that is best represented by graphs of exponential or logarithmic functions, and explain the reasoning. TOP: Modelling data using logarithmic functions
KEY: logarithmic function | regression function | extrapolate

10. ANS: C PTS: 1 DIF: Grade 12 REF: Lesson 7.5
OBJ: 6.4 Graph data and determine the exponential or logarithmic function that best approximates the data. | 6.5 Interpret the graph of an exponential or logarithmic function that models a situation, and explain the reasoning. | 6.6 Solve, using technology, a contextual problem that involves data that is best represented by graphs of exponential or logarithmic functions, and explain the reasoning. TOP: Modelling data using logarithmic functions
KEY: logarithmic function | regression function | curve of best fit

SHORT ANSWER

1. ANS:
3

PTS: 1 DIF: Grade 12 REF: Lesson 7.2
OBJ: 4.1 Express a logarithmic equation as an exponential equation and vice versa. | 4.2 Determine the value of a logarithmic expression, such as log_2 8, without technology. | 4.5 Determine the approximate value of a logarithmic expression, such as log_2 9, with technology.
TOP: Evaluating logarithmic expressions KEY: logarithmic function

2. ANS:
$x = 2.367$

PTS: 1 DIF: Grade 12 REF: Lesson 7.4
OBJ: 5.2 Determine the solution of an exponential equation in which the bases are not powers of one another. | 5.3 Solve problems that involve the application of exponential equations to loans, mortgages and investments.
TOP: Solving exponential equations using logarithms KEY: logarithmic function

PROBLEM

1. ANS:

Use the compound interest formula with $P = 1600$ and $i = \frac{0.03}{12}$ or 0.0025.

$$A = 1600(1 + 0.0025)^n$$

$$A = 1600(1.0025)^n$$

Solve for n when $A = 2000$:

$$2000 = 1600(1.0025)^n$$

$$\frac{2000}{1600} = (1.0025)^n$$

$$\log\left(\frac{2000}{1600}\right) = \log(1.0025)^n$$

$$\log(1.25) = n\log(1.0025)$$

$$\frac{\log(1.25)}{\log(1.0025)} = n$$

$$n = 89.368\ldots$$

Convert the number of months to years:

$$\frac{89.368\ldots}{12} = 7.447\ldots$$

In the eighth year after the initial investment, the investment will reach $2000.

PTS: 1 DIF: Grade 12 REF: Lesson 7.4
OBJ: 5.2 Determine the solution of an exponential equation in which the bases are not powers of one another. | 5.3 Solve problems that involve the application of exponential equations to loans, mortgages and investments.
TOP: Solving exponential equations using logarithms KEY: logarithmic function

2. ANS:

Use the half-life equation with $A_0 = 122$, $A = 6.64$, and $t = 21.0$.

$$A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

$$6.64 = 122\left(\frac{1}{2}\right)^{\frac{21.0}{h}}$$

Solve for h:

$$6.64 = 122\left(\frac{1}{2}\right)^{\frac{21.0}{h}}$$

$$\frac{6.64}{122} = \left(\frac{1}{2}\right)^{\frac{21.0}{h}}$$

$$\log\left(\frac{6.64}{122}\right) = \log\left(\frac{1}{2}\right)^{\frac{21.0}{h}}$$

$$\log\left(\frac{6.64}{122}\right) = \frac{21.0}{h}\log\left(\frac{1}{2}\right)$$

$$h = \frac{21.0\log\left(\frac{1}{2}\right)}{\log\left(\frac{6.64}{122}\right)}$$

$$h = 5.000\ldots$$

The half-life of bismuth-210 is 5.0 days.

PTS: 1 DIF: Grade 12 REF: Lesson 7.4
OBJ: 5.2 Determine the solution of an exponential equation in which the bases are not powers of one another. | 5.3 Solve problems that involve the application of exponential equations to loans, mortgages and investments.
TOP: Solving exponential equations using logarithms
KEY: logarithmic function | half-life