

Measurement

Suggested Time: 15 Hours

Unit Overview

Focus and Context

The focus of this unit is on the use of SI and imperial units. Students will determine linear measures using a variety of referents, and will find the surface area and volume of 3-D objects. They will use proportional reasoning to convert a measurement within or between SI and imperial systems. Although students have previously worked with a variety of SI units, this is their first exposure to imperial units. As with other problem solving situations, it is important that proper estimation and measurement strategies be used and that error analysis be an integral part of the learning process.

Math Connects

The vast majority of countries now use the SI system of measurement. The most notable exception is the United States in which imperial units are used extensively. Its close proximity and strong economic ties to Canada makes the US a destination of choice for leisure, work and business, so it is important that people who visit and trade with the US be comfortable with the imperial system.

Students are exposed to both SI and imperial units. We typically measure our personal height in feet and inches. Our weight is often measured in pounds, although at a hospital it is often recorded in kilograms. Prints are sold in sizes based on inches and the most popular paper sizes, for example, letter and legal, are sized in imperial units. Sports such as golf measure distance in yards and fabric is often sold in yards or metres.

The ability to convert between metric and imperial units is an important skill for many Canadians. Many technical diagrams and building codes in our construction industry contain metric units. Tradespersons must be able to convert to the imperial measures when ordering materials such as dimensional lumber, sheathing, and fasteners. Some quantities are measured in both SI and imperial units in certain industries. For example, flooring can be measured in square yards or square meters and cement in cubic yards or cubic meters. Workers in such industries must be able to perform area and volume calculations using both types of units with equal efficiency.

Geometry with 3-D objects will help prepare students for success in future secondary and post-secondary courses and for work in industries where spatial ability is important. In order to increase cost effectiveness, employees determine the surface area and volume of objects with speed and accuracy and with minimum wastage.

Process Standards

[C] Communication
 [CN] Connections
 [ME] Mental Mathematics
 and Estimation

[PS] Problem Solving
 [R] Reasoning
 [T] Technology
 [V] Visualization

SCO Continuum

Grade 9	Grade 10	Grade 11
Shape and Space	Measurement	
not addressed	<p>M1 Solve problems that involve linear measurement, using:</p> <ul style="list-style-type: none"> • SI and imperial units of measure • estimation strategies • measurement strategies. <p>[ME, PS, V]</p>	not addressed
not addressed	<p>M2 Apply proportional reasoning to problems that involve conversions between SI and imperial units of measure.</p> <p>[C, ME, PS]</p>	<p>2200 (Measurement) M2 Solve problems that involve scale diagrams, using proportional reasoning.</p> <p>[CN, PS, R, V]</p>
<p>9SS2 Determine the surface area of composite 3-D objects to solve problems.</p> <p>[C, CN, PS, R, V]</p>	<p>M3 Solve problems, using SI and imperial units, that involve the surface area and volume of 3-D objects, including:</p> <ul style="list-style-type: none"> • right cones • right cylinders • right prisms • right pyramids • spheres. <p>[CN, PS, R, V]</p>	<p>2200 (Measurement) M3 Demonstrate an understanding of the relationships among scale factors, area, surface areas and volumes of similar 2-D shapes and 3-D objects.</p> <p>[C, CN, PS, R, V]</p>

Measurement

Outcomes

Students will be expected to

M1 Solve problems that involve linear measurement, using:

- **SI and imperial units of measure**
- **estimation strategies**
- **measurement strategies.**

[ME, PS, V]

Elaborations—Strategies for Learning and Teaching

International System of Units, officially called the *Système International d'Unités*, or SI, is based on the metric system. It is the primary system of measurement used throughout the world and in science. In the SI system, length is always measured using a base unit of metres. This system is used because it is convenient and logical. It uses prefixes indicating powers of 10. The prefix kilo, for example, is represented by 10^3 . The imperial system, however, is not based on the base ten system. In the imperial system, length may be measured using inches, feet, yards or miles. While the SI system of measure is the official measurement system in Canada, students will have some exposure to and experience with imperial units of measure.

Worldwide, there are many instances where the SI system is used instead of the imperial system. For example, speed limits in many countries are posted in kilometres per hour instead of miles per hour. In most science laboratories, researchers use SI units. Sporting events such as the Olympics use metric measurements as well (100 m sprint versus 100 yd. sprint).

There are other instances where the imperial system is used. For example, in the construction industry, lengths are measured in inches (2 in. x 4 in.). Sports, such as football and golf, measure distance using yards and marathons are measured in miles.

There are also cases where both systems are used. For example, wrenches come in imperial and metric sizes. Snowmobiles made in the United States require imperial sized wrenches, whereas snowmobiles made elsewhere would use metric sized wrenches. The price of groceries is often given in both imperial (price per pound) and metric (price per 100 g).

Students were introduced to metre and centimetre as a unit of measurement in Grade 3(3SS3). This was extended to include millimetre and kilometre in Grade 5(5SS2). This is the first time that imperial units are formally addressed in the K-12 curriculum.

General Outcome: Develop spatial sense and proportional reasoning

Suggested Assessment Strategies*Journal*

- Ask students to respond to the following questions:
 - (i) Explain the difference between SI and imperial units.
 - (ii) If you were to measure someone's height would you use SI or imperial units? Why?
 - (iii) Explain why the unit yards is used instead of metres in football and golf.

(M1)

Paper and Pencil

- Ask students to explore the historical origin of various SI units and imperial units such as the metre and the yard.

(M1)

Resources/Notes**Authorized Resource***Foundations and Pre-Calculus
Mathematics 10*

Measurement

Outcomes

Students will be expected to

M1 Continued...

Achievement Indicators:

M1.1 *Provide referents for linear measurements, including millimetre, centimetre, metre, kilometre, inch, foot, yard and mile, and explain the choices.*

M1.2 *Compare SI and imperial units, using referents.*

M1.3 *Estimate a linear measure, using a referent, and explain the process used.*

M1.4 *Justify the choice of units used for determining a measurement in a problem solving context.*

Elaborations—Strategies for Learning and Teaching

Students should be given an opportunity to reflect on and discuss the following:

- What is measurement?
- How do we measure things?
- What are some commonly used units of measurement?

A referent is an object that can be used to help estimate a measurement. From the earliest introduction to metric units, students have had experience relating non-standard and standard units of measurement. They have used referents to estimate the length of an object in centimetres, metres and millimetres. The millimetre, for example, is suitable for measuring small distances.

Estimation activities help students focus on the attribute being measured and help develop familiarity with the measuring unit. Using referents, students should explore both SI and imperial units to measure objects.

Some common referents for linear measurement include:

Referent	Description
1 mm	thickness of a dime, thickness of a fingernail
1 cm	width of a fingernail, width of black keys on a standard piano, width of a crayon, width of a paper clip
1 m	distance from a doorknob to the floor, width of a volleyball net
1 km	distance you can walk comfortably in 15 minutes
1 in.	thickness of a hockey puck, length from end of thumb to first knuckle
1 ft.	length of a standard floor tile
1 yd.	distance from the tip of the nose to the outstretched fingers, average length of a guitar
1 mi.	distance you can walk comfortably in 20 minutes

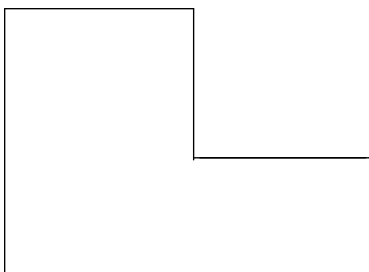
General Outcome: Develop spatial sense and proportional reasoning

Suggested Assessment Strategies

Paper and Pencil

- Ask students to list household objects that are:
 - (i) 2 ft. long
 - (ii) 4 in. thick
 - (iii) 12 cm wide

(M1.1, M1.3)
- Ask students to complete the following:
 - (i) Estimate the perimeter of the figure in an appropriate SI unit.



- (ii) If all the angles are right angles, is it necessary to measure all sides of the figure?

(M1.3)

Journal

- Ask students to respond to the following:
 - (i) A hockey net is 6 ft. wide. Explain how you could use a personal referent to mark off a width of approximately 6 ft.?

(M1.1, M1.3)
 - (ii) What referent could you use to estimate how much snow fell after a snowstorm? Explain your choice.

(M1.1, M1.3, M1.4)

Resources/Notes

Authorized Resource

Foundations and Pre-Calculus Mathematics 10

1.1: Imperial Measures of Length
1.2: Math Lab: Measuring Length and Distance

Teacher Resource (TR): pp.8-14
Master 1.1a, 1.2

DVD:

Smart Lesson 1.1, 1.2

Student Book (SB): pp.4-15

Preparation and Practice Book (PB): pp.2-11

Web Link

This video describes imperial and SI units using referents.

www.youtube.com/watch?v=DQPQ_q59xyw&feature=youtube

The classroom clip demonstrates students estimating linear measurements using referents.

www.k12pl.nl.ca

Measurement

Outcomes

Students will be expected to

M1 Continued...

Achievement Indicators:

M1.1, M1.2 *Continued*
M1.3, M1.4 *Continued*

Elaborations—Strategies for Learning and Teaching

The thumb can also be used as a measurement device. A person's measurement could be determined by measuring the distance around his or her thumb using string. Twice around the thumb is equal to once around the wrist. Twice around the wrist is once around the neck. Twice around the neck is once around the waist. Students should be encouraged to experiment with this.

Each referent is a suggestion. Encourage students to select their own personal referent that makes sense to them. Students may, for example, use their waist height as a referent for one metre. If they determine the height of the seat of a chair to be approximately half of their waist height, then the seat of the chair is 0.5 metres high. If students are determining the length of a room, for example, they could count the tiles on the floor since the length of a standard tile is 1 foot in length. Students should be given an opportunity to use their referents to provide estimations for various items and justify their choice of unit.

General Outcome: Develop spatial sense and proportional reasoning

Suggested Assessment Strategies

Performance

- Students could estimate the length of the following objects using a referent and explain how they determined their answer.
 - (i) height of a door
 - (ii) width of a smartboard or whiteboard
 - (iii) length of a keyboard
 - (iv) height of a light switch
 - (v) height of an electrical outlet

(M1.1, M1.3)

- Students could participate in a school scavenger hunt to locate the following:
 - (i) an object 2 dimes thick
 - (ii) an object 3 inches long
 - (iii) an object 4 sub sandwiches long
 - (iv) an object located 5 guitars away

(M1.1, M1.3)

Resources/Notes

Authorized Resource

Foundations and Pre-Calculus Mathematics 10

1.1: Imperial Measures of Length

1.2: Math Lab: Measuring Length and Distance

TR: pp.8-14

Master 1.1a, 1.2

DVD:

Smart Lesson 1.1, 1.2

SB: pp.4-15

PB: pp.2-11

Measurement

Outcomes

Students will be expected to

M1 Continued...

Achievement Indicators:

M1.5 *Solve problems that involve linear measure, using instruments such as rulers, calipers or tape measures.*

M1.6 *Describe and explain a personal strategy used to determine a linear measurement; e.g., circumference of a bottle, length of a curve, perimeter of the base of an irregular 3-D object.*

Elaborations—Strategies for Learning and Teaching

Students should use instruments such as rulers, calipers, and tape measures to determine lengths of objects using both SI and imperial units. Incorporating a variety of measuring devices and using shapes that interest students will provide more meaningful learning experiences. Ask students to give some examples of measuring instruments that are commonly used to measure distance in the home or workplace. Students can explain or demonstrate how one of these instruments work. They could brainstorm a list of objects, use their referents to estimate the length and then actually measure the items to check their accuracy. The information could be displayed in the form of a poster or a paper foldable.

Students should be given the opportunity to explore the environment to develop these measurement skills. Consider the following examples:

- What is the length of the parking lot?
- How far do you have to walk from the bus to the school's entrance?
- How much fencing is needed or used to enclose the school grounds?
- What is the circumference of a school bus tire?
- What is the length of a window?
- What is the height of a locker?

Students should also be exposed to strategies when estimating the length of a curve or measuring irregular shaped objects such as a computer mouse, a horseshoe magnet, a badminton racquet, etc. Using string and rulers, encourage students to measure, for example, the circumference of a circular clock. Students may determine the circumference by laying the string around the clock and then measuring the length of the string. This would be a good opportunity to extend this activity to estimate the length of a curve or a portion of the circular object without the use of string. Using a ruler, students could measure the distance between the endpoints of the curve. Since the shortest distance between two points is a straight line, their estimate of the length of the curve would be greater than this distance. Remind students to describe the process used to determine their answer.

General Outcome: Develop spatial sense and proportional reasoning

Suggested Assessment Strategies

Performance

- Ask students to estimate using a referent and then measure using a measuring tool:
 - (i) Within your school, how far must you walk from the library to the gymnasium?
 - (ii) What are the dimensions of the technology lab?
 - (iii) What is the width of a computer screen?
 - (iv) What is the width of the staircase?
 - (v) What is the perimeter of an iPod?
 - (vi) What is the perimeter of the base of the recycling bin?

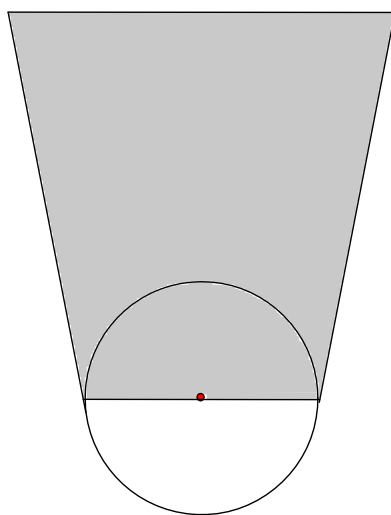
(M1.1, M1.3, M1.5, M1.6)

- Ask students to estimate the total distance they would walk during a typical day at school. In groups of two or three, students can use referents to explore their individual routes throughout the building for a particular day in their school cycle. Once completed, they can compare their results with other students in the class and discuss the strategies used to obtain their distances.

(M1.1, M1.3, M1.4, M1.5)

Journal

- Ask students to respond to the following:
 - (i) Describe how you would determine the circumference of a cross section of a basketball going through the centre of the basketball. State the referent, the unit, and the measuring instrument used. (M1.6)
 - (ii) Explain a strategy that Julie can use to determine the perimeter of the basketball key represented by the shaded region. (M1.6)



Resources/Notes

Authorized Resource

Foundations and Pre-Calculus Mathematics 10

1.1: Imperial Measures of Length

1.2: Math Lab: Measuring Length and Distance

TR: pp.8-14

Master 1.1a, 1.2

DVD:

Smart Lesson 1.1, 1.2

SB: pp.4-15

PB: pp.2-11

Measurement

Outcomes

Students will be expected to

M2 Apply proportional reasoning to problems that involve conversions between SI and imperial units of measure.

[C, ME, PS]

Achievement Indicators:

M2.1 *Using proportional reasoning, convert a measurement within or between SI and imperial systems.*

M2.2 *Solve a problem that involves the conversion of units within or between SI and imperial units.*

M2.3 *Verify, using unit analysis, a conversion within or between SI and imperial systems, and explain the conversion.*

Elaborations—Strategies for Learning and Teaching

Students applied proportional reasoning to problems involving ratios in Grade 8 (8N5) and similar polygons in Grade 9 (9SS3).

A proportion is a statement where two ratios are equal. For example,

$$\frac{100 \text{ cm}}{1 \text{ m}} = \frac{350 \text{ cm}}{3.5 \text{ m}}$$

A portion of an SI unit is written in decimal form. A decimal system is used for several reasons. It can immediately be seen which of the two values is greater and it is easier to do calculations involving decimal numbers. A portion of an imperial unit, however, is generally written in fractional form. Many rulers, for example, are marked with imperial units showing one inch divided into eighths or sixteenths.

Students sometimes have difficulty working with fractions. For example, students often have difficulty multiplying a fraction by a whole number. Although they have worked with operations involving fractions in intermediate grades (7N5, 8N6, 9N3), this would be a good opportunity to review these concepts prior to completing any conversions.

To convert from one measurement system to another, students need to understand the relationship between the units of length in each system. Depending on the context of the problem, students will determine when conversions should be exact or when it might be appropriate to use an approximate conversion. For example, 1 in. is exactly 2.54 cm or approximately 2.5 cm. Conversions between SI and imperial units should be limited to commonly used linear units of measure. Consider the following:

- cm ↔ in.
- m ↔ ft.
- mi. ↔ km

Unusual conversions such as miles ↔ mm should be avoided.

General Outcome: Develop spatial sense and proportional reasoning

Suggested Assessment Strategies

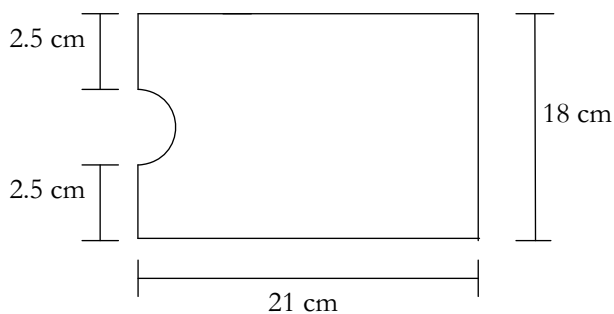
Paper and Pencil

- Ask students to answer the following:
 - (i) Convert the following measurements.
 - (a) 2.5 m = ___km
 - (b) 8 in. = ___ ft.
 - (c) 7 m = ___ in.

(M2.1)
 - (ii) A road sign says to turn left in 1000 feet. Approximately how far is this distance in kilometres?

(M2.1, M2.2)

- Ask students to complete the following:
 - (i) Calculate the perimeter of the following. Express your answer to the nearest tenth of an inch.



(M2.1, M2.2)

- (ii) Which distance below is the longest?
0.7in., 1000yd., 1km, 910m

(M2.1, M2.2)

- Many display tags on items in a store show the SI and imperial measurements. Students could make a display tag for a rectangular picture having dimensions 32 in. by 50 in. illustrating the dimensions of the picture in inches and in centimetres.

(M2.1, M2.2)

Journal

- Ask students to explain when it might be appropriate to use an approximate conversion. Give an example.

(M2.1, M2.2)

Resources/Notes

Authorized Resource

Foundations and Pre-Calculus Mathematics 10

1.1: Imperial Measures of Length 1.3: Relating SI and Imperial Units

TR: pp.8-11, 15-18

Master 1.1a, 1.3

DVD:

Animations 1.1, 1.3

Smart Lesson 1.1, 1.3

SB: pp.4-12, 16-25

PB: pp.2-8, 12-21

Measurement

Outcomes

Students will be expected to

M2 Continued...

Achievement Indicators:

M2.1, M2.2, M2.3 *Cont'd*

M2.4 *Justify, using mental mathematics, the reasonableness of a solution to a conversion problem.*

Elaborations—Strategies for Learning and Teaching

When performing various conversions, students could use a multiplicative method. In order to convert a measurement in feet to inches, for example, multiply the length in feet by 12. Proportional reasoning is another method students will use when converting between units. Technology may be used, where appropriate, to solve proportions.

When converting within or between SI and imperial units, unit analysis should be used to verify that units in a conversion are correct. Consider the following example:

Convert 200 metres to kilometres.

$$200 \text{ m} \times \frac{1 \text{ km}}{1000 \text{ m}}$$

0.200 km

It is important for students to notice that the units have changed but the actual distance has not. The distance of 200 m is the same as 0.200 km.

Prompt students to discuss the value of the ratio $\frac{1 \text{ km}}{1000 \text{ m}}$?

Since 1 km = 1000 m, the ratio $\frac{1 \text{ km}}{1000 \text{ m}} = 1$. Students should recognize when a numerical value is multiplied by 1, the value remains the same.

This is the basis of unit conversion.

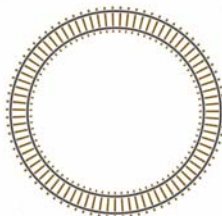
Students should be encouraged to check the reasonableness of their answers when performing conversions. Does the answer make sense? Did the student overestimate or underestimate? In this example students should realize when converting 200 m to km, the answer should be less than 200.

General Outcome: Develop spatial sense and proportional reasoning

Suggested Assessment Strategies

Paper and Pencil

- Ask students to answer the following:
 - Robert bought a Lego™ railway track for his birthday. When he assembled the full track, the distance the toy train travelled along the inside of the track was 182.4 cm.



Calculate the radius of the inside edge of the track to the nearest millimetre. Estimate the radius of the outside edge of the track.

(M1.3, M2.2)

- A carpenter would like to place trim around a rectangular window measuring 41 in. by 27 in. If the trim costs \$1.92/ft., what is the approximate cost of the trim for the window before taxes? Verify the conversions using unit analysis. (M2.1, M2.2, M2.3)

- Convert the following measurement showing unit analysis.

$$6 \text{ yd.} = \underline{\hspace{1cm}} \text{ cm}$$

(M2.1, M2.3)

Journal

- Linda and her friends go for a 15 minute walk. At the end of their walk they wonder how far they travelled in feet, metres, kilometres and miles. Explain how you can estimate this distance and show how to express the answer in the various units.

(M2.1, M2.2)

Resources/Notes

Authorized Resource

Foundations and Pre-Calculus Mathematics 10

1.1: Imperial Measures of Length

1.3: Relating SI and Imperial Units

TR: pp.8-11, 15-18

Master 1.1a, 1.3

DVD:

Animations 1.1, 1.3

Smart Lesson 1.1, 1.3

SB: pp.4-12, 16-25

PB: pp.2-8, 12-21

Measurement

Outcomes

Students will be expected to

M3 Solve problems, using SI and imperial units, that involve the surface area and volume of 3-D objects, including:

- right cones
- right cylinders
- right prisms
- right pyramids
- spheres

[CN, PS, R, V]

Achievement Indicators:

M3.1 *Sketch a diagram to represent a problem that involves surface area or volume.*

M3.2 *Determine the surface area of a right cone, right cylinder, right prism, or a right pyramid, using an object or its labelled diagram.*

Elaborations—Strategies for Learning and Teaching

In Grade 7, students developed an understanding of two-dimensional measurement. Students calculated the area of triangles, circles and parallelograms (7SS2). In Grade 8, students determined the surface area and volume of three-dimensional objects limited to right rectangular prisms, right triangular prisms and right cylinders (8SS3, 8SS4). The focus was on gaining a conceptual understanding of surface area through the use of nets rather than on the use of formulae. Volume was calculated using $V = (\text{Base Area}) \times \text{height}$. Students also applied the Pythagorean theorem to determine the length of a side in a right triangle (8SS1). In Grade 9, work with surface area was extended to include composite 3-D objects (9SS2). In this unit, students will extend the concepts of surface area and volume to include right pyramids, right cones, and spheres. It is important for students to understand how the surface area and volume formulas of 3-D objects are developed to prevent memorization of formulas.

Understanding how to calculate surface area can be helpful in many real world applications. For example, surface area can be used to estimate the amount of paint needed to paint a house or to know how much wrapping is needed to cover a container. Throughout this unit students should be encouraged to draw diagrams to help them visualize the 3-D objects that are described.

In Grade 8, students used nets to calculate the surface area of right rectangular prisms, right triangular prisms, and right cylinders. They were also exposed to regular polygons. These topics should be reviewed before determining the surface area formula of a right pyramid and right cone.

A pyramid is a 3-D figure with a polygon base. The shape of the base determines the name of the pyramid. Students should be exposed to a right pyramid with a triangular base, square base, and a rectangular base. Various websites display the nets of the different types of pyramids to give students a visual. When the base of a right pyramid is a regular polygon, the triangular faces are congruent (regular pyramid).

General Outcome: Develop spatial sense and proportional reasoning

Suggested Assessment Strategies

Performance

- In groups of 2 or 3, students will explore the various nets of geometric solids, form a 3-D solid and create a mobile. Ask students to determine the various surface area formulas by examining the individual faces of the net. Encourage students to write down the formula representing the area of each face prior to cutting out the net. Students can then form the 3-D solid, develop the formula to represent the surface area and it will become one piece of a mobile. Repeat for each solid. The mobile should include a right cone, right cylinder, right prism and a right pyramid.

(M3.2)

Paper and Pencil

- Ask students to answer the following:
 - A water tank is the shape of a right circular cylinder 30 ft. long and 8 ft. in diameter. How many square feet of sheet metal was used in its construction?
(M3.1, M3.2)
 - A rectangular prism measuring 12 cm by 8 cm by 5 cm has a cube with side length 3 cm removed from its corner. Determine its surface area.
(M3.1, M3.2)
 - A tent that has a square base with a side length of 9 ft. and a height of 7.5 ft. needs a canvas cover. Determine the amount of canvas needed to cover the tent. (Hint: The floor of the tent is not made of canvas.)
(M3.1, M3.2)

Resources/Notes

Authorized Resource

Foundations and Pre-Calculus Mathematics 10

1.4: Surface Areas of Right Pyramids and Right Cones

TR: pp.19-22

Master 1.1b, 1.1c, 1.4-1.6

DVD:

Animation 1.4

Smart Lesson 1.4

SB: pp.26-35

PB: pp.22-30

Web Link

Nets can be generated at the following website:

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=205>

Measurement

Outcomes

Students will be expected to

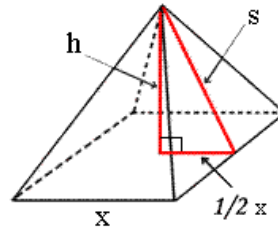
M3 Continued...

Achievement Indicators:

M3.1, M3.2 *Continued*

Elaborations—Strategies for Learning and Teaching

Students will have to differentiate between the height of a right pyramid and its slant height. The height refers to the perpendicular height from the apex to the base, h , and the slant height is the altitude of the triangular face, s . A common student misconception is relating the slant height to the edge of the pyramid. The Pythagorean theorem will be used to calculate the slant height given the perpendicular height and the appropriate base dimension, x , or given the edge length from the apex to the base and the appropriate base dimension.



Encourage students to calculate the surface area of a right pyramid using nets. They should recognize the surface area is the sum of the base and four triangular faces.

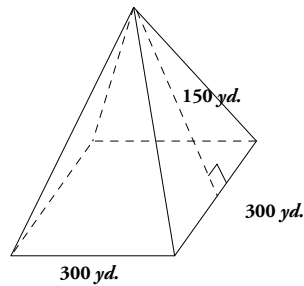
Students should be given an opportunity to apply the surface area of a right pyramid in real world problems. For example, students can compare the surface area of two right square pyramids with different dimensions to determine which pyramid requires more glass to enclose its space.

General Outcome: Develop spatial sense and proportional reasoning

Suggested Assessment Strategies

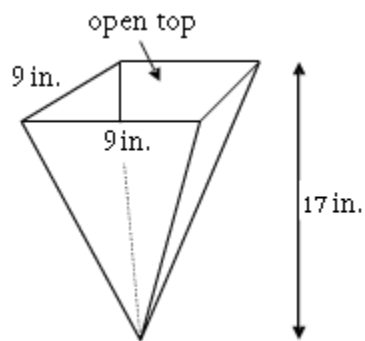
Paper and Pencil

- Ask students to determine the surface area of the object shown.



(M3.2)

- Ask students to answer the following:
John hangs 6 flower pots around his house. The flower pots need to be painted. How much paint, in square feet, would John need to paint the outside of the flower pots?



(M3.2)

Resources/Notes

Authorized Resource

Foundations and Pre-Calculus Mathematics 10

1.4: Surface Areas of Right Pyramids and Right Cones

TR: pp.19-22

Master 1.1b, 1.1c, 1.4-1.6

DVD:

Animation 1.4

Smart Lesson 1.4

SB: pp.26-35

PB: pp.22-30

Web Link

An interactive site demonstrating surface area and volume of 3-D objects and its nets.

<http://www.learner.org/interactives/geometry/area.html>

http://www.bgfl.org/bgfl/custom/resources_ftp/client_ftp/ks2/maths/3d/index.htm

<http://www.sadlier-oxford.com/math/enrichment/gr4/EN0411b/EN0411b.htm>

Measurement

Outcomes

Students will be expected to

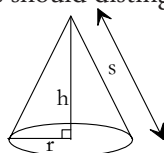
M3 Continued...

Achievement Indicators:

M3.1, M3.2 *Continued*

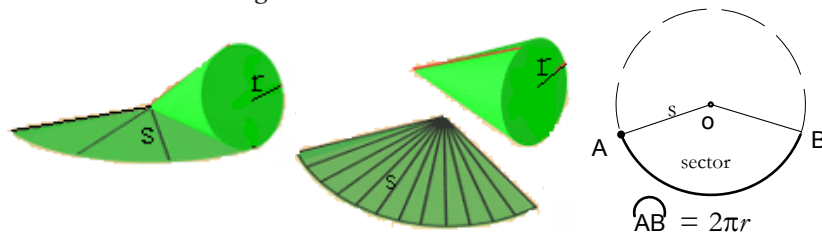
Elaborations—Strategies for Learning and Teaching

The surface area of a cone consists of the area of the circular base and the curved surface. Students should distinguish between the height and slant height of a cone.



Students are familiar with the area formula of a circle, πr^2 . They have not been exposed to the area of the curved surface of a cone, therefore, this would be a good opportunity for students to model this area.

Students lay a cone on its side on a flat surface. Rolling it one complete turn will form a sector of a circle. They should notice the area of the curved surface of the cone is equal to the area of the sector. Although students were exposed to circles, circumference and arcs in Grade 9, the terms *sector* and *arclength* are new and should be defined.



where r is the radius of the cone

Students should be given an opportunity to discover the following:
What part of the cone would be the radius of the sector? What is the relationship between the circumference of the base of the cone and the arclength of the sector?

To find the area of the curved surface of the cone, teachers compare the sector of the circle to the whole circle using proportional reasoning.

$$\frac{\text{area of curved surface of the cone}}{\text{area of circle}} = \frac{\text{circumference of base of cone}}{\text{circumference of circle}}$$

$$\frac{\text{area of sector of circle}}{\text{area of circle}} = \frac{\text{arclength of sector (portion of the circumference) of the circle}}{\text{circumference of circle}}$$

$$\frac{\text{area of sector of circle}}{\pi s^2} = \frac{2\pi r}{2\pi s}$$

The area of the curved surface of the cone, which is equal to the area of the sector of the circle, simplifies to πrs .

Therefore, the surface area formula of a right cone is represented by:

$$\text{Surface Area} = \pi rs + \pi r^2$$

General Outcome: Develop spatial sense and proportional reasoning

Suggested Assessment Strategies

Paper and Pencil

- Ask students to answer the following:
 - (i) Mary has made about 10 conical party hats out of cardboard. How much cardboard was used in total if each hat has a radius of 14 cm and a slant height of 25 cm?
(M3.1, M3.2)
 - (ii) Tyler works at a local ice cream parlor making waffle cones. If a finished cone is 6 in. high and has a base diameter of 4 in., what is the surface area of the cone (not including the area of the base)?
(M3.1, M3.2)
 - (iii) A right cone has a circular base with a diameter 29 cm and a height of 38 cm. Calculate the surface area of the cone to the nearest tenth of a square centimetre.
(M3.1, M3.2)

Resources/Notes

Authorized Resource

Foundations and Pre-Calculus Mathematics 10

1.4: Surface Areas of Right Pyramids and Right Cones

TR: pp.19-22

Master 1.1b, 1.1c, 1.4-1.6

DVD:

Animation 1.4

Smart Lesson 1.4

SB: pp.26-35

PB: pp.22-30

Measurement

Outcomes

Students will be expected to

M3 Continued...

Achievement Indicator:

M3.3 Determine an unknown dimension of a right cone, right cylinder, right prism, or right pyramid, given the object's surface area and the remaining dimensions.

Elaborations—Strategies for Learning and Teaching

Question students on how to find the surface area of a cone if the radius and the height are given, but the slant height is unknown. Students should first draw diagrams to help them organize their information and then apply the Pythagorean theorem. Remind students that problems involving multi-step calculations should be rounded in the final step.

Students should be given the opportunity to explore the surface area of objects in the environment. Examples may include an ice cream cone, a water fountain cup, a can of soup, a hockey puck, or the packaging of a Toblerone bar.

Once students have become fluent in determining the surface area of 3-D objects, they will then determine an unknown dimension. Students are not expected to rearrange formulas at this level. They should first substitute the given information into the formula and then solve for the unknown. In the case where both the slant height and surface area are given, students will not be expected to find the radius of a right cone. Similarly, students will not be expected to solve for the radius of a cylinder when the height and surface area are given. These types of problems would require an application of the quadratic formula which will not be introduced until M2201/M2200.

General Outcome: Develop spatial sense and proportional reasoning

Suggested Assessment Strategies

Paper and Pencil

- Ask students to answer the following:
 - (i) The surface area of a right cone is 125 in.^2 and its radius is 4.7 in. What is the slant height of the right cone?
(M3.3)
 - (ii) A cylinder has a surface area of 412 cm^2 . The height is three times greater than the radius. Approximate the height of the cylinder.
(M3.3)
 - (iii) A right pyramid has a surface area of 154 cm^2 . A right cone has a base radius of 3 cm. The cone and pyramid have equal surface area. What is the height of the cone to the nearest tenth of a centimetre?
(M3.3)

Resources/Notes

Foundations and Pre-Calculus Mathematics 10

1.4: Surface Areas of Right Pyramids and Right Cones

TR: pp.19-22

Master 1.1b, 1.1c, 1.4-1.6

DVD:

Animation 1.4

Smart Lesson 1.4

SB: pp.26-35

PB: pp.22-30

Measurement

Outcomes

Students will be expected to

M3 Continued...

Achievement Indicators:

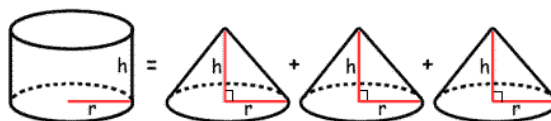
M3.4 Determine the volume of a right cone, right cylinder, right prism, or a right pyramid using an object or its labelled diagram.

M3.5 Describe the relationship between the volumes of:

- right cones and right cylinders with the same base and height
- right pyramids and right prisms with the same base and height.

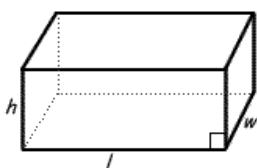
Elaborations—Strategies for Learning and Teaching

Students will investigate the relationship between the volume of a right cone and a right cylinder and between the volume of a right pyramid and a right prism. The volume formula of a right cylinder ($V = \pi r^2 h$) was developed in Grade 8. The volume of a right cone with the same base and the same height as a cylinder should be developed through investigation. One activity involves students filling three congruent cones, all having the same height and radii as a cylinder, with water. Pour the water from the cones into the cylinder.



Students should see that the water from the three cones fills the cylinder entirely. This means it takes the volume of three cones to equal one cylinder. Looking at this in reverse, each cone is one-third the volume of a cylinder. Hence, the volume of a right cone is represented by: $V = \frac{1}{3} \pi r^2 h$

Prisms are named according to the shape of their base such as a triangular prism or rectangular prism. The volume of a right rectangular prism was developed in Grade 8 using the formula $V = (\text{base area}) \times \text{height}$. Students will continue to use this formula for any right prism. For example:



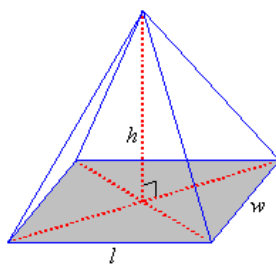
Volume of a right rectangular prism is given by:

$$V = (\text{base area}) \times \text{height}$$

$$V = (\text{Area of rectangle}) \times h$$

$$V = l \times w \times h$$

The volume of a right pyramid is found by calculating one third of the volume of its related right prism. A rectangular pyramid, for example, can be filled with rice and students can calculate how many times it takes to fill a rectangular prism having the same base length, base width and height.



Volume of a right rectangular pyramid is given by:

$$V = \frac{1}{3} (\text{base area}) \times \text{height}$$

$$V = \frac{1}{3} lwh$$

General Outcome: Develop spatial sense and proportional reasoning

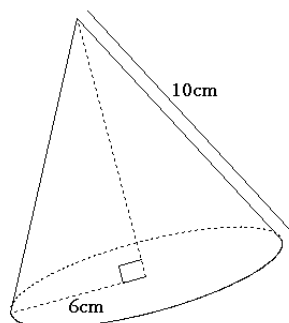
Suggested Assessment Strategies

Paper and Pencil

- Ask students to answer the following:

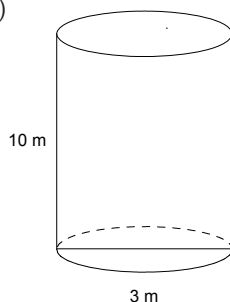
(i) Determine the volume of the following:

(a)



(M3.4)

(b)



(ii) A cake decorating bag is in the shape of a cone. To the nearest cubic centimetre, how much frosting will fit into the bag if the diameter is 15 cm and the height is 25 cm?

(M3.4)

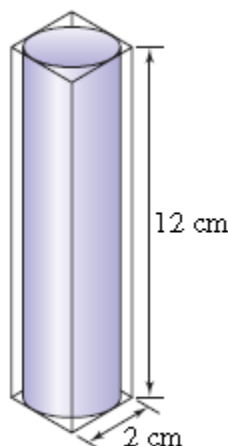
(iii) A cone and a cylinder have the same height and the same base radius. If the volume of the cylinder is 81 cm^3 , what is the volume of the cone in cm^3 ? Explain.

(M3.4, M3.5)

(iv) Find the volume of a square based pyramid where the length of each base side and the height measures 2.7 ft.

(M3.4)

(v) A closed cylindrical can is packed in a box. What is the volume of the empty space between the can and the box?



(M3.4)

Resources/Notes

Authorized Resource

Foundations and Pre-Calculus Mathematics 10

1.5: Volumes of Right Pyramids and Right Cones

TR: pp.23-25

Master 1.1d

DVD:

Dynamic Activity 1.5

Smart Lesson 1.5

SB: pp.36-44

PB: pp.31-39

Web Link

A powerpoint presentation providing information and visuals of right prisms.

[http://www.worldofteaching.com/powerpoints/maths/Right%20Prism%20volume.ppt#256,1,RIGHT PRISM](http://www.worldofteaching.com/powerpoints/maths/Right%20Prism%20volume.ppt#256,1,RIGHT%20PRISM)

The professional learning site provides a classroom clip of students demonstrating the relationship between the volume of a right cone and right cylinder and a right pyramid and right prism with the same base and height.

www.k12pl.nl.ca

Measurement

Outcomes

Students will be expected to

M3 Continued...

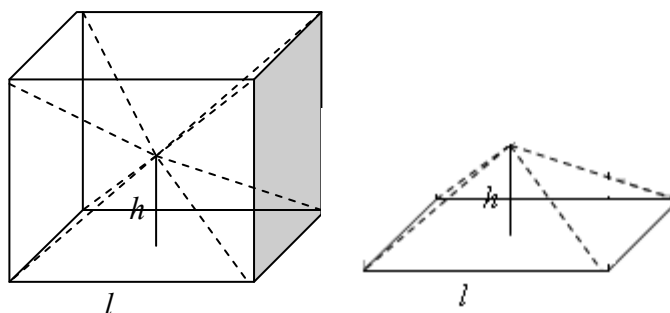
Achievement Indicators:

M3.4 Continued

Elaborations—Strategies for Learning and Teaching

As an alternative, the volume formula of a right pyramid could be developed using a symbolic approach. Consider the following:

- To find the volume of a right rectangular pyramid, start with a cube having side length l . Segments are drawn from each vertex that travels through the cube's center to other vertices. In doing so, six equal pyramids are created.



- The volume of a cube can be found using the formula for a prism, $V = (\text{base area}) \times \text{height}$. All sides of a cube are equal, resulting in $V = (l \times l) \times l = l^3$. Students have not yet been exposed to exponent laws with variable bases. Teachers could introduce and explain the relationship $l^2 \times l = l^3$ for this particular application. The volume for each of the pyramids is equal. Therefore, the volume of each pyramid must be $\frac{1}{6}$ the original cube's volume. Thus, $V_{\text{pyramid}} = \frac{1}{6} l^3$.
- The base length of the cube is equal to two pyramid heights, $l = 2h$. Therefore the formula can be generalized further. Through substitution,

$$V = \frac{1}{6} l^3$$

$$V = \frac{1}{6} l^2 l$$

$$V = \frac{1}{6} l^2 (2h)$$

$$V = \frac{2}{6} l^2 h$$

$$V = \frac{1}{3} (\text{Area of base}) \times \text{height}$$

As students work through problems involving the volume of 3-D objects, they will also have to determine an unknown dimension. Although students are not expected to rearrange formulas at this level, they will have to take the square root of both sides of the equation when solving for the radius in a right cone or cylinder. Students should substitute given information into the formula and then solve for the unknown. They will not be expected, however, to find the length of a cube given the volume since students are not familiar with cube roots. This will be explored later in the Roots and Powers unit.

M3.6 Determine an unknown dimension of a right cone, right cylinder, right prism, or right pyramid, given the object's volume and the remaining dimensions.

General Outcome: Develop spatial sense and proportional reasoning

Suggested Assessment Strategies

Paper and Pencil

- Ask students to answer the following:
 - (i) A cone has a volume of 30 cm^3 and a base area of 15 cm^2 . What is the height of the cone? (M3.6)
 - (ii) A cylinder has a volume of 132.6 cm^3 and a height of 8.5 cm . What is the diameter of the cylinder? (M3.6)
 - (iii) A cord of firewood is 128 cubic feet. Jan has 3 storage bins for firewood that each measure 2 ft. by 3 ft. by 4 ft. Does she have enough storage space to hold a full cord of firewood? Explain. (M3.4)

Journal

- Ask students to respond to the following:
The Department of Transportation and Works would like to determine the volume of road salt they have in a stock pile near the Burin Peninsula junction. What are some possible ways to determine the volume of the pile, assuming it is in the shape of a right cone. (M3.4, M3.5)

Performance

- Ask students to explore volume of 3-D objects with the same base and height. Connections can be made by using containers and water or by using folding geometric solids. Students can make connections between:
 - (i) a right cylinder and a right cone
 - (ii) a rectangle and a right rectangular pyramid. (M3.4, M3.5)

Resources/Notes

Authorized Resource

Foundations and Pre-Calculus Mathematics 10

1.5: Volumes of Right Pyramids and Right Cones

TR: pp.23-25

Master 1.1d

DVD:

Dynamic Activity 1.5

Smart Lesson 1.5

SB: pp.36-44

PB: pp.31-39

Measurement

Outcomes

Students will be expected to

M3 Continued...

Achievement Indicators:

M3.7 Determine the surface area and volume of a sphere, using an object or its labelled diagram.

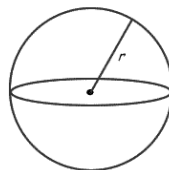
Elaborations—Strategies for Learning and Teaching

Students should be given opportunities to discuss real-life examples of spheres in their surroundings. Examples might include a tennis ball, a marble, a basketball, and a globe.

Review the terms radius and diameter as well as the area formula of a circle. An orange, for example, approximates a sphere and the area of its peel represents the surface area of the sphere. When introducing the surface area formula of a sphere, a good activity is using an orange.

- Find the greatest distance around an orange (around the centre of the orange) using string. Use this as the circumference of a cross section through the centre of the orange.
- Using this measurement, find the radius of the sphere. ($C = 2\pi r$)
- Use a compass to draw 6 circles with that radius.
- Peel the orange and fill as many circles as completely as you can.
- Students should fit exactly 4 circles.

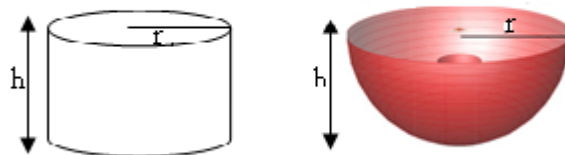
The surface area of a sphere is the area of the 4 circles. Therefore,



$$SA = 4\pi r^2$$

When introducing the volume formula of a sphere, the following activity using a cylinder and a hemisphere having the same height and radius, would be beneficial.

- Cut a small basketball in half, creating a hemisphere.
- Obtain a cylinder having the same radius and height of the hemisphere.
- When the hemisphere is fully filled with rice, place it into the cylinder. It will fill $\frac{2}{3}$ of the cylinder.



The volume of a cylinder is $V = \pi r^2 h$. Therefore, the volume of the hemisphere would be $V = \frac{2}{3} (\pi r^2 h)$. The height is also the radius of the hemisphere resulting in $V = \frac{2}{3} \pi r^3$. The volume of a sphere is twice the volume of a hemisphere which results in $V = \frac{4}{3} \pi r^3$.

General Outcome: Develop spatial sense and proportional reasoning

Suggested Assessment Strategies

Paper and Pencil

- Ask students to answer the following:
 - (i) An official basketball has a radius of 12.5 cm and usually has a leather covering. Approximately how much leather, in cm^2 , is required to cover 12 official basketballs?
(M3.7)
 - (ii) A carnival clown has 75 m^3 of helium compressed in a tank. How many spherical balloons with a radius of 0.25 m can be filled with the helium from the tank?
(M3.7)
 - (iii) Eight basketballs are put in a container. The radius of each basketball is 10 cm. If the container is shaped like a square based pyramid, approximately how much room will be left (volume space not occupied by a basketball) if each side of the base measures 40 cm and the height is 70 cm?
(M3.4, M3.7)
 - (iv) A spherical Christmas ornament measures 12 cm in circumference. What is the approximate volume of the cubed box that will hold this ornament?
(M3.4, M3.7)
 - (v) A heavy sphere with diameter 20 cm is dropped into a right circular cylinder with a base radius of 10 cm and a height of 34 cm.
 - (a) If the cylinder is half full of water, what is the total volume of the water and the sphere?
 - (b) How high will the water rise once the sphere is completely under the water? (Note: Once the sphere is dropped into the water, the water level will rise to a height that represents the volume of the water plus the volume of the sphere.)
(M3.4, M3.7, M3.6)

Resources/Notes

Authorized Resource

Foundations and Pre-Calculus Mathematics 10

1.6: Surface Area and Volume of a Sphere

TR: pp.26-29

DVD:

Animation 1.6

Smart Lesson 1.6

SB: pp.45-54

PB: pp.40-49

Web Link

The classroom clip posted on the professional learning site demonstrates students developing the formula for the surface area and volume of a sphere.

www.k12pl.nl.ca

Measurement

Outcomes

Students will be expected to

M3 Continued...

Achievement indicators:

M3.8 *Determine an unknown dimension of a sphere, given the object's surface area and the remaining dimensions.*

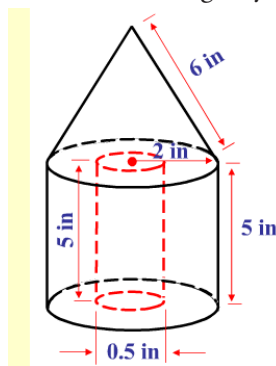
M3.9 *Solve a problem that involves surface area or volume, using an object or its labelled diagram of a composite 3-D object.*

Elaborations—Strategies for Learning and Teaching

Students will determine an unknown radius of a sphere given the surface area by taking the square root of both sides of the equation. Students are not expected to determine an unknown radius of a sphere given the volume because they are not familiar with cube roots. This will be explored in the Unit: Roots and Powers. Discussions surrounding volume and missing measures, however, may be beneficial. When a sporting goods company, for example, makes a racquetball, they know the required volume of the racquetball and can use the volume formula to determine the radius.

Students determined the surface area of composite objects involving right prisms and right cylinders in Grade 9. They will now determine the surface area and volume of composite objects made from right prisms, right pyramids, right cylinders, right cones, and spheres. When determining surface area, encourage students to keep the context of the problem in mind. For example, to determine the surface area of a straw, the shape of which is a cylinder, the top and bottom would not be included.

When decomposing a composite object, encourage students to look for component parts such as right cones, right cylinders, right prisms, right pyramids and spheres. The focus here is for students to recognize that when calculating the surface area of a composite object, the area of overlap must be taken into consideration. The volume is determined by adding and subtracting the volumes independent of the overlap. In the following diagram, for example, the volume of the composite object can be determined by subtracting the volume of the inside cylinder from the total volume of the cone and the larger cylinder.



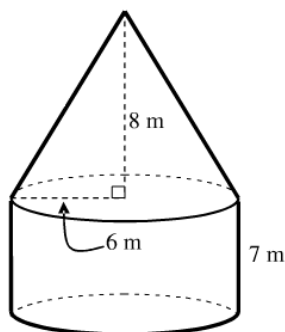
Students should be given opportunities to explore the surface area and volume of composite objects in the environment.

General Outcome: Develop spatial sense and proportional reasoning

Suggested Assessment Strategies

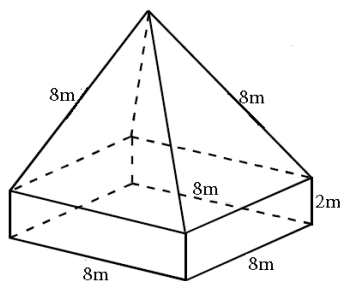
Paper and Pencil

- Ask students to answer the following:
 - A sphere has a surface area of 80 in^2 . Determine the diameter. (M3.8)
 - Determine how much paint is needed to paint the outside of the following composite 3-D object.



(M3.2, M3.9)

- Calculate the volume of the following figure:



(M3.4, M3.9)

Performance

- Ask students to combine three or more shapes, including prisms, pyramids and cylinders to make an interesting sculpture. Use appropriate units, metric or imperial, to label its dimensions and find the total surface area and volume of the sculpture.

(M3.1, M3.2, M3.4, M3.9)

Resources/Notes

Authorized Resource

Foundations and Pre-Calculus Mathematics 10

1.6: Surface Area and Volume of a Sphere

TR: pp.26-29

DVD:

Animation 1.6

Smart Lesson 1.6

SB: pp.45-54

PB: pp.40-49

1.7: Solving Problems Involving Objects

TR: pp.30-32

DVD:

Animation 1.7

Smart Lesson 1.7

SB: pp.55-61

PB: pp.50-58

