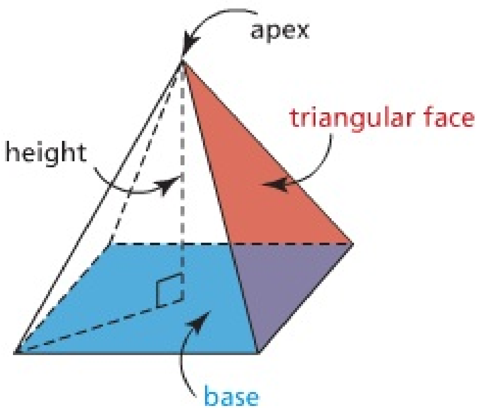
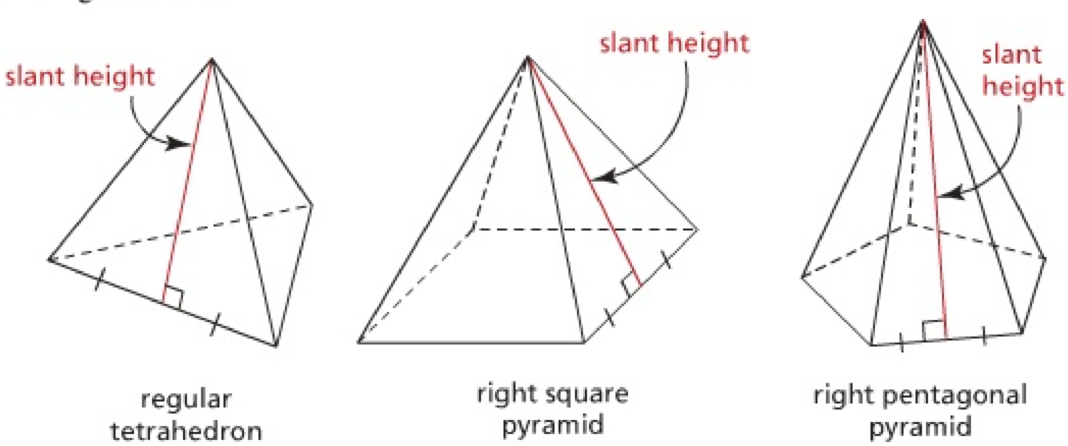
Section 1-4 Surface area of Right Pyramids and Right Cones

(DON'T LOSE!) Name:_________ Date:_______

A **right pyramid** is a 3-dimensional object that has triangular faces and a base that is a polygon. The shape of the base determines the name of the pyramid. The triangular faces meet at a point called the **apex**. The *height* of the pyramid is the perpendicular distance from the apex to the centre of the base.

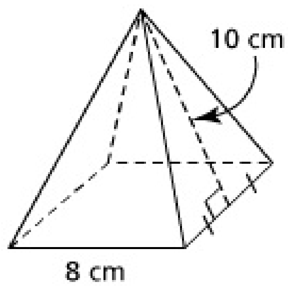


When the base of a right pyramid is a regular polygon, the triangular faces are congruent. Then the **slant height** of the right pyramid is the height of a triangular face.

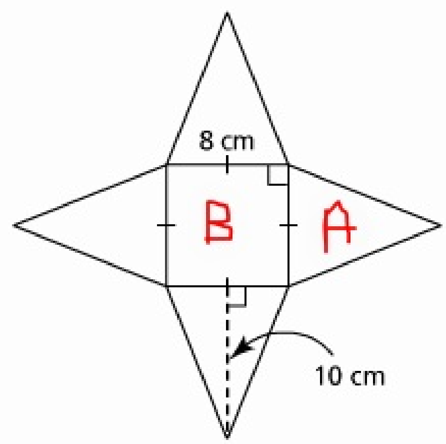


The surface area of a right pyramid is the sum of the areas of the triangular faces and the base.

Ex) Determine the surface area of a right square pyramid that has a slant height of 10 cm and a base with each side being 8 cm.



This net shows the faces and base of the pyramid.



The area, A, of each triangular face is:

$A = \frac{1}{2}(8)(10)$

$A = 40$

The area, B, of the base is:

$B = (8)(8)$

$B = 64$

So, the surface area, SA, of the pyramid is:

$SA = 4A + B$

$SA = 4(40) + 64$

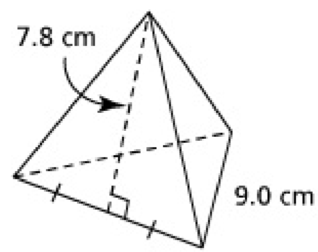
$SA = 224$

The surface area of the pyramid is 224 cm^2.

Ex 2

Determining the Surface Area of a Regular Tetrahedron Given Its Slant Height

Jeanne-Marie measured then recorded the lengths of the edges and slant height of this regular tetrahedron. What is its surface area to the nearest square centimetre?



SOLUTION

The regular tetrahedron has 4 congruent faces. Each face is a triangle with base 9.0 cm and height 7.8 cm.

The area, A, of each face is:

$$A = \frac{1}{2}(9.0 \text{ cm})(7.8 \text{ cm})$$

The surface area, SA, is:

$$SA = 4\left(\frac{1}{2}\right)(9.0 \text{ cm})(7.8 \text{ cm})$$

$$SA = 140.4 \text{ cm}^2$$

The surface area of the tetrahedron is approximately 140 cm^2.

When finding the surface area of a regular of a pyramid it is important to ALWAYS have the slant height. This will involve the use of the Pythagorean Theorem!!!

"In any right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs."

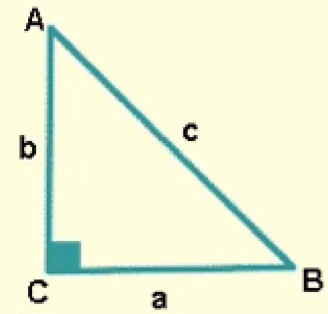
This relationship can be stated as:

$$c^2 = a^2 + b^2$$

for any right triangle

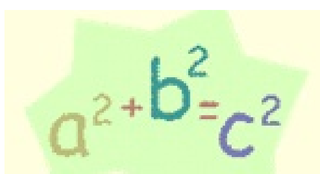
and is known as the

Pythagorean Theorem.

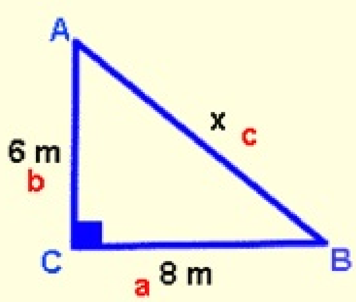


a, b are legs.
c is the hypotenuse
(c is across from the right angle).

Also you can use:



Example 1:



Find x.

Answer: 10 m

$$c^2 = a^2 + b^2$$

$$x^2 = 64 + 36$$

$$x^2 = 100$$

$$\sqrt{x^2} = \sqrt{100}$$

$$x = 10$$

If we know (6,8,10) are Pythagorean triples we know others as well:

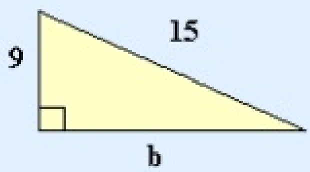
IE (3,4,5).....(18, 24, 30) and (30, 40, 50) are all Pythagorean triples [they fit the sides of a right triangle]

I the Pythagorean triple (5, 12, 13)

A) Hypothenuse = B) Legs are

C) three more Pythagorean triples would be: [, ,] [, ,] [, ,]

Example: Solve this triangle.



$$a^2 + b^2 = c^2$$

$$9^2 + b^2 = 15^2$$

$$81 + b^2 = 225$$

Take 81 from both sides:

$$b^2 = 144$$

$$b = \sqrt{144}$$

$$\mathbf{b = 12}$$

Pythagorean Triple is:

DOM (Definitely on Midterm....definitely on the first exam) rectangular based pyramids

Ex 1)

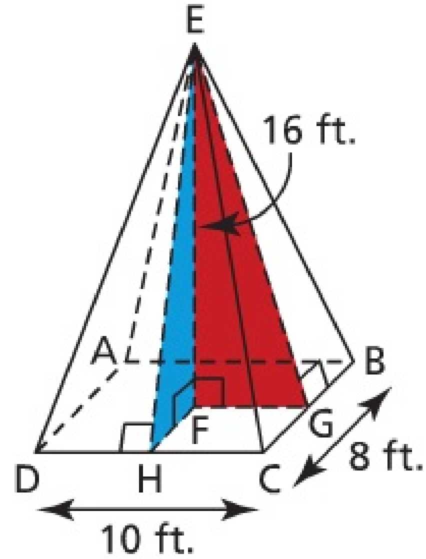
A right rectangular pyramid has base dimensions 8 ft. by 10 ft., and a height of 16 ft. Calculate the surface area of the pyramid to the nearest square foot.

Draw it (Sketch)

There are 4 triangular faces and a rectangular base.

Sketch the pyramid and label its vertices. Opposite triangular faces are congruent.

Draw the heights on two adjacent triangles.



In $\triangle EFH$, FH is $\frac{1}{2}$ the length of BC, so FH is 4 ft.

EF is the height of the pyramid, which is 16 ft.

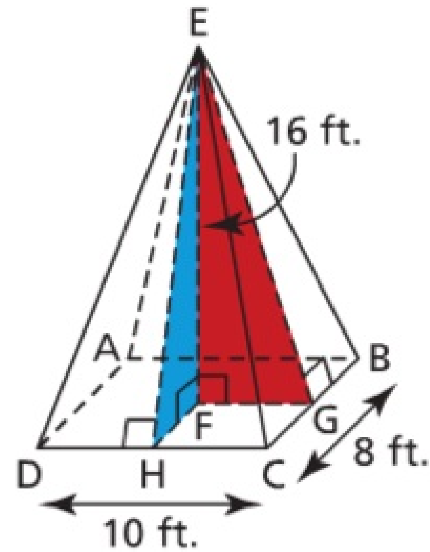
Note: are all faces the same size? Why not?____________________________

EF = Slant or the Apex Height? __________________

How can you get EH (Slant height of ________ face)?

How can you get EG(Slant Height of __________ face?

Surface Area   In English



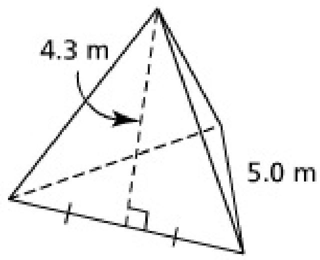
Solution: SA (Mathematically)

SA = Area of [Front and Back Face] + Area of [Left and Right Face] + Area of [Base]

SA =

Practice exercises:

1. Calculate the surface area of this regular tetrahedron to the nearest square metre.



Note: are you given the slant height?

Yes or no

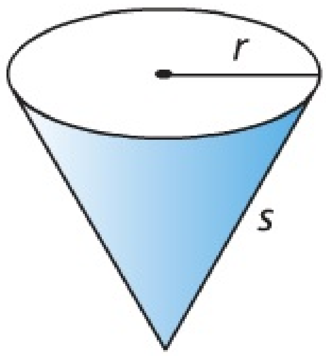
Method: compute the area of each triangular face and multiply your answer by 4.

2 A square based pyramid has a 16 x 16 ft. base with an apex height of 6 ft. Sketch a picture of the pyramid. Determine the slant height of each face and use it to compute the surface area of the pyramid including the base.

3. A right rectangular pyramid has base dimensions 4 m by 6 m, and a height of 8 m. Calculate the surface area of the pyramid to the nearest square metre.

Sketch the pyramid! Find the slant height of each face then compute the surface area.

Surface Area of a Cone



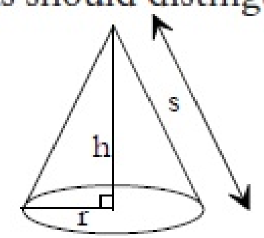
radius = ________= ________

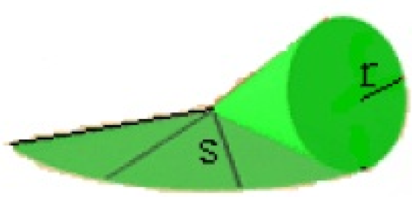
diameter = d = 2x______ = ______

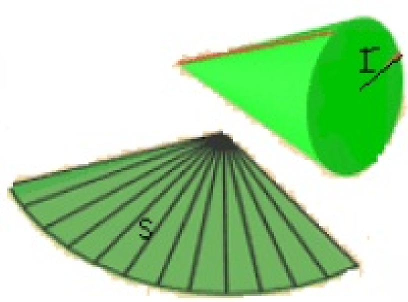
s = ___________

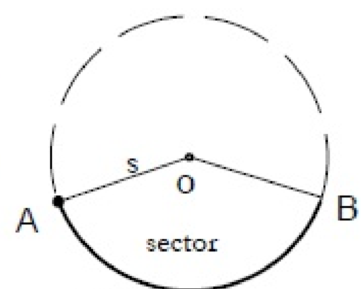
h =____________

The surface area of a cone consists of the area of the circular base and the curved surface. Students should distinguish between the height and slant height of a cone.



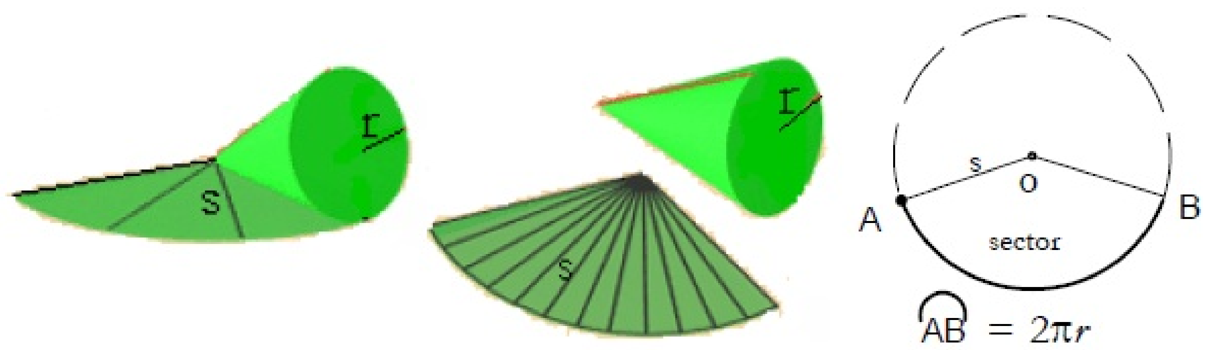






$\overset{\frown}{AB} = 2\pi r$

where r is the radius of the cone



where r is the radius of the cone

$$\frac{\text{area of curved surface of the cone}}{\text{area of circle}} = \frac{\text{circumference of base of cone}}{\text{circumference of circle}}$$

$$\frac{\text{area of sector of circle}}{\text{area of circle}} = \frac{\text{arclength of sector (portion of the circumfererence) of the circle}}{\text{circumference of circle}}$$

$$\frac{\text{area of sector of circle}}{\pi s^2} = \frac{2\pi r}{2\pi s}$$

The area of the curved surface of the cone, which is equal to the area of the sector of the circle, simplifies to πrs.

Therefore, the surface area formula of a right cone is represented by:
Surface Area = $\pi rs+\pi r^2$

What do I need to remember?

The surface area of the Cone is = Lateral Area (Area of the Curved surface) + Area of a circle.

= $\pi rs + \pi r^2$ where $\pi = 3.14$ r = radius s = slant

apex height I never used in the formula.

Right Cones Continued

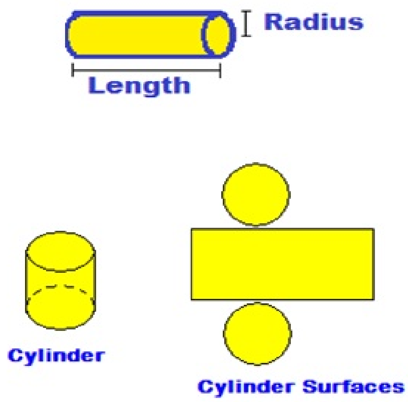
Ex) Mary has made about 10 conical party hats out of cardboard. How much cardboard was used in total if each hat has a radius of 14 cm and a slant height of 25 cm?

Question students on how to find the surface area of a cone if the radius and the height are given, but the slant height is unknown. Students should first draw diagrams to help them organize their information and then apply the Pythagorean theorem. Remind students that problems involving multi-step calculations should be rounded in the final step.

Ex) Tyler works at a local ice cream parlor making waffle cones. If a finished cone is 6 in. high and has a base diameter of 4 in., what is the surface area of the cone (not including the area of the base)?

Ex) A right cone has a circular base with a diameter 29 cm and a height of 38 cm. Calculate the surface area of the cone to the nearest tenth of a square centimetre.

The Surface area of a right Cylinder (Done in Grade 8!) 1-4



Surface area = 2xcircles + area of the rectangle

length of the rectangle = l = _________

width of the rectangle = h = __________

Surface Area = _______ + _________

2 circles   area of the side (rectangle)

Ex)

A water tank is the shape of a right circular cylinder 30 ft. long and 8 ft. in diameter. How many square feet of sheet metal was used in its construction?

Ex) The diameter of a certain tomato soup can is 4in. If the height is 8 in., determine the surface area of the can to the nearest inch.

Practice Exercise: Page 34 5, 7, 8, 9 a, b (Surface area only), 13 (Final Exam Q) Do On Loose leaf and STAPLE to these NOTES

M3.3 *Determine an unknown dimension of a right cone, right cylinder, right prism, or right pyrmaid, given the object's surface area and the remaining dimensions.*

This means if you are given the surface area, you have to find either the radius, diameter, or height of the right object.

Once students have become fluent in determining the surface area of 3-D objects, they will then determine an unknown dimension. Students are not expected to rearrange formulas at this level. They should first substitute the given information into the formula and then solve for the unknown. In the case where both the slant height and surface area are given, students will not be expected to find the radius of a right cone. Similarly, students will not be expected to solve for the radius of a cylinder when the height and surface area are given. These types of

Ex1)

The surface area of a right cone is 125 $in.^2$ and its radius is 4.7 in. What is the slant height of the right cone?

Ex2) The surface area of a cylinder is 2000 cm^2 . Its radius is 25 cm. Algebraically determine the height of the cylinder to the nearest tenth.

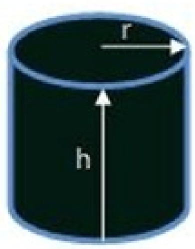
Ex3) The Surface area of a cone is 5000 $in.^2$ If the diameter of the cone is 20 in., determine the slant height of the cone to the nearest one hundredth.

Section 1-5

> M3.4 *Determine the volume of a right cone, right cylinder, right prism, or a right pyramid using an object or its labelled diagram.*

Volume of any object is always defined as :____________ x ____________

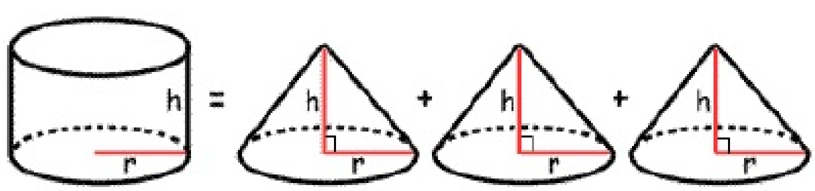
1) Volume of a right cylinder.



Remember that the radius and the height must be in the same units - convert them if necessary. The resulting *volume* will be in those cubic units. So if the height and radius are both in centimeters, then the *volume* will be in cubic centimeters.

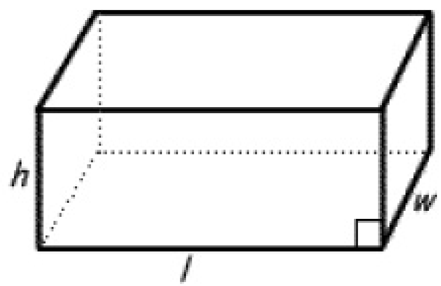
The *volume of a cylinder* is found by multiplying the area of its top or base by its height and is defined as: $V = \pi \cdot r^2 \cdot h$

2 Volume of a Right Cone: Since it take three cones to fill a cylinder with the same height and radius V = one third of a cylinder: $V = \frac{1}{3}\pi r^2 h$



Students should see that the water from the three cones fills the cylinder entirely. This means it takes the volume of three cones to equal one cylinder. Looking at this in reverse, each cone is one-third the volume of a cylinder. Hence, the volume of a right cone is represented by: $V=\frac{1}{3}\pi r^2 h$

3 Volume of a right prism:



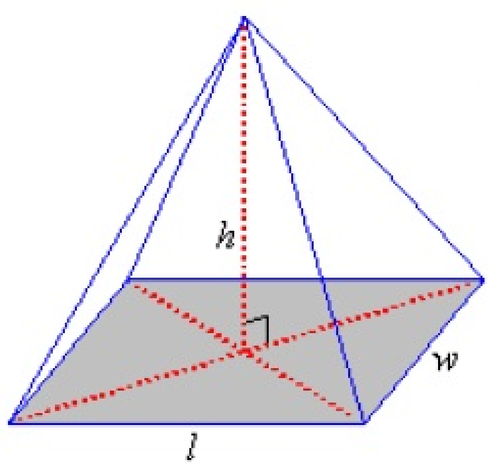
Volume of a right rectangular prism is given by:

V = (base area) × height.

V = (Area of rectangle) × h

V = $l \times w \times h$

4 The volume of a right pyramid: Is always one third that of a right prism.



Volume of a right rectangular pyramid is given by:

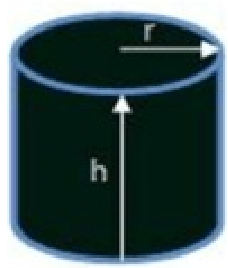
$$V = \frac{1}{3}\text{(base area)} \times \text{height}$$

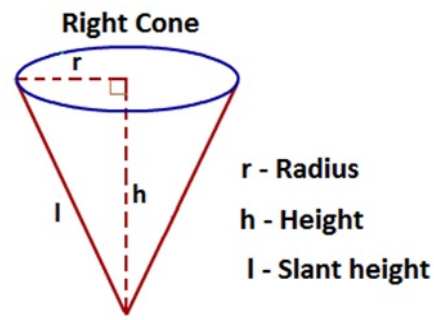
$$V = \frac{1}{3} lwh$$

Summary:

1 Volume of right cylinder

Related Object: the right cone





V =

SA =

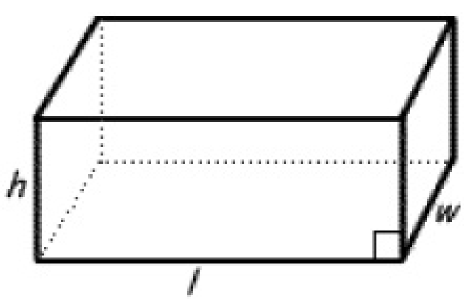
V =

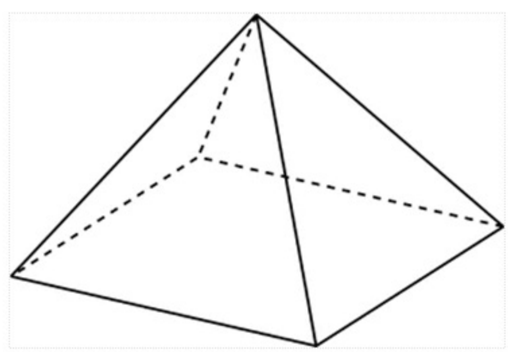
SA =

NOTE: r and h are equal for both figures!

2 Volume of right Prism

Related Object: the right rectangular (or square based) pyramid





V =

SA =

V =

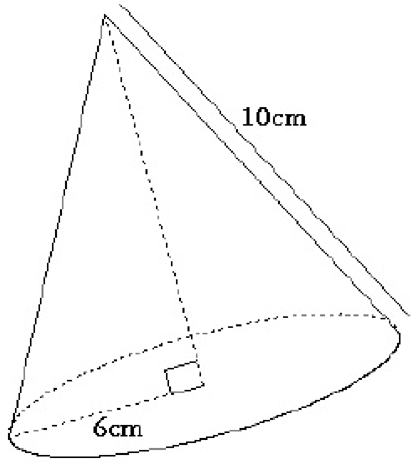
SA =

Note: l, w and h are equal in both figures

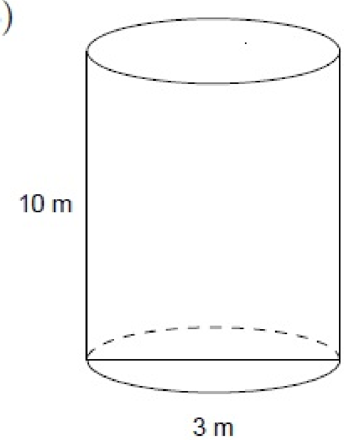
Examples:

(i) Determine the volume of the following:

(a)



(b)



(M3.4)

(ii) A cake decorating bag is in the shape of a cone. To the nearest cubic centimetre, how much frosting will fit into the bag if the diameter is 15 cm and the height is 25 cm?

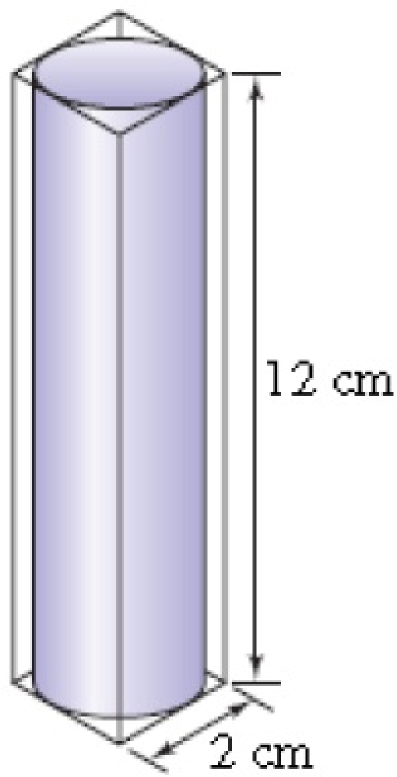
Practice:

1) A cone and a cylinder have the same height and the same base radius. If the volume of the cylinder is 81 cm^3, what is the volume of the cone in cm^3? Explain.

2) Find the volume of a square based pyramid where the length of each base side and the height measures 2.7 ft.

3) A closed cylindrical can is packed in a box. What is the volume of the empty space between the can and the box?

(M3.4)



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1-5

M3.6 Determine an unknown dimension of a right cone, right cylinder, right prism, or right pyramid, given the object's volume and the remaining dimensions.

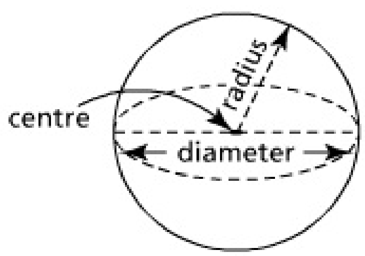
Ex1) A cone has a volume of 30 cm^3 and a base area of 15 cm^2. What is the height of the cone?

EX2) A cylinder has a volume of 132.6 cm^3 and a height of 8.5 cm. What is the diameter of the cylinder?

Ex3) A cord of firewood is 128 cubic feet. Jan has 3 storage bins for firewood that each measure 2 ft. by 3 ft. by 4 ft. Does she have enough storage space to hold a full cord of firewood? Explain.

1-7 Surface Area and Volume of a Sphere Section 1-7 Text

A **sphere** is the set of points in space that are the same distance from a fixed point, which is the *centre*. A line segment that joins the centre to any point on the sphere is a *radius*. A line segment that joins two points on a sphere and passes through the centre is a *diameter*.



Surface Area:

Activity for Homework:

1) Peel an orange (or a grape fruit) (Very small pieces approximately 1 cm by 1 cm if you can)

2) Draw 6 circles on a sheet that have the same diameter as your orange

3) Determine how many circles you can fill with the small peels of your orange.

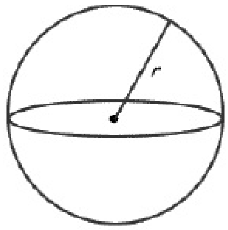
4) TAKE A PICTURE of your circles that are filled with your name clearly visible in the picture.

5) Show me the picture in class. (Do not have a phone or digital camera?? See me I can help :-))

How many circles did you fill? _______ What is the area of each circle? ________ (Formula)

Surface area of a sphere is: SA = __________

Ex1) The surface area of a sphere is the area of the 4 circles. Therefore,



$SA = 4\pi r^2$

The diameter of a baseball is approximately 3 in. Determine the surface area of a baseball to the nearest square inch.

Ex 2) **SOLUTION**

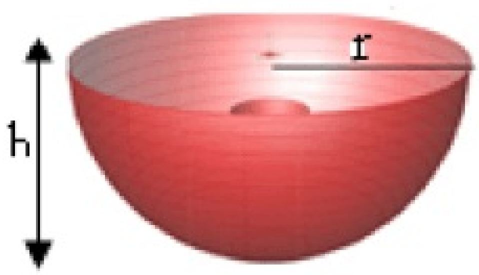
An official basketball has a radius of 12.5 cm and usually has a leather covering. Approximately how much leather, in cm^2, is required to cover 12 official basketballs?

$SA = 4\pi r^2$ Substitute $r = 1.5$.

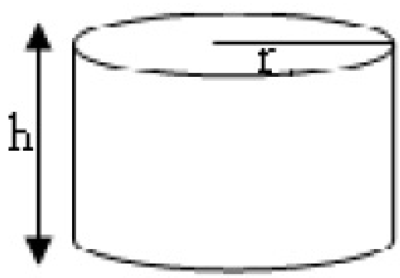
Ex) The surface area of a tennis ball is 120 cm^2. Determine the radius and diamter of the ball to the nearest cm?

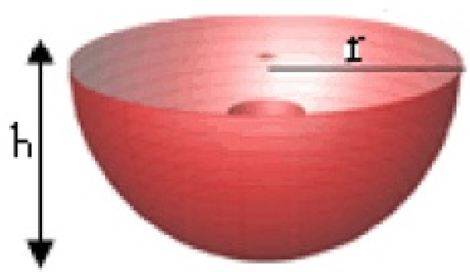
Volume of a Sphere

Hemisphere: basically is half of a sphere



When the hemisphere is fully filled with rice, place it into the cylinder. It will fill $\frac{2}{3}$ of the cylinder.





Therefore Vol of HALF of a Sphere is: 2/3 of a cylinder (NOTE h = r)

$$Vol_{hemisphere} = \frac{2}{3} cylinder = \frac{2}{3}\pi^2 rh$$

=FILL in Mistakes notes h = r and pi is squared

$$Vol_{full\ Sphere} = 2x\left(\frac{2}{3} cylinder\right) = \frac{4}{3}\pi^2 rh$$

Ex) The Volume of a ball that has a radius of 8 in. Is: _____

Volume = _________

Ex 1 A carnival clown has 75 m^3 of helium compressed in a tank. How many spherical balloons with a radius of 0.25 m can be filled with the helium from the tank?

Ex 2

Eight basketballs are put in a container. The radius of each basketball is 10 cm. If the container is shaped like a square based pyramid, approximately how much room will be left (volume space not occupied by a basketball) if each side of the base measures 40 cm and the height is 70 cm?

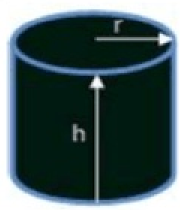
Ex 3

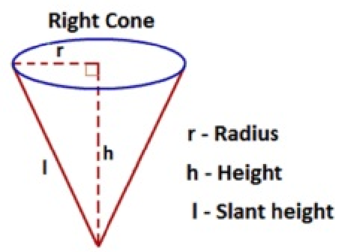
A heavy sphere with diameter 20 cm is dropped into a right circular cylinder with a base radius of 10 cm and a height of 34 cm.

(a) If the cylinder is half full of water, what is the total volume of the water and the sphere?

Review of All formulas to date:

Names:





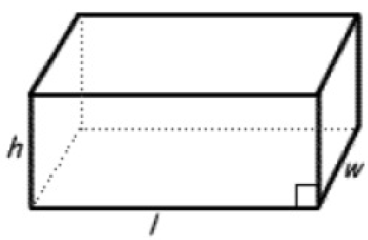
V=

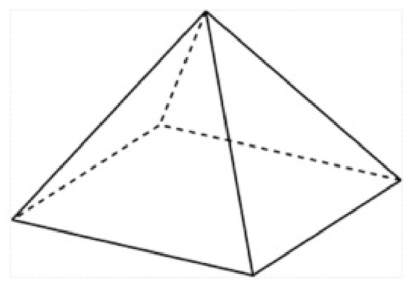
SA =

V=

SA =

Names :





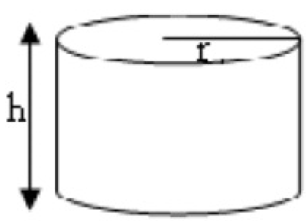
V=

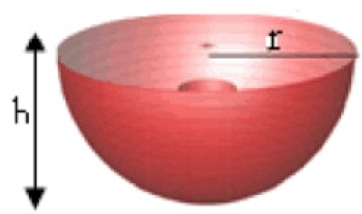
SA =

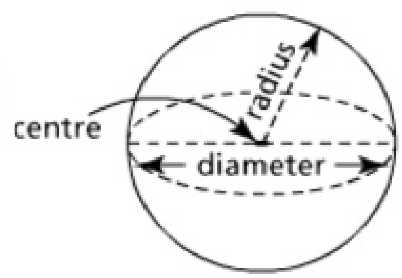
V=

SA =

Names:







V=

V=

V=

SA=

Practice: Page 51 3, 4, 7, 9, 16

Applications of Volume and Surface are (Composite objects!)