Name: $\qquad$
Teacher: $\qquad$

# Mathematics 3201 

## PRE-PUBLIC EXAMINATION <br> JUNE 2014

Value: 100 Marks
Duration: 3 Hours

General Instructions
This examination consists of two parts. Part I of the exam consists of selected response questions and Part II consists of constructed response questions.

## Selected Response (50 marks)

Select the letter of the correct response from those provided. Place the CAPITAL LETTER of the correct response in the blank space provided on your Selected Response Answer Sheet.

Constructed Response (50 marks)
Answer ALL questions fully and concisely in the space provided. Show all work.

## Student Checklist

The items below are your responsibility. Please ensure that they are completed.

- Write your name and teacher's name on the top of this page.
- Write your name, teacher's name, course name and number on the Selected Response Answer Sheet.
- Check the exam to ensure there are no missing pages. There are $\underline{\mathbf{2 0}}$ pages in total.

ALL MATERIALS MUST BE PASSED IN WITH THIS EXAM. Use your time wisely.
Good luck!
$\qquad$

## PART I

## Total Value: 50 marks

## Answer all items. Place the CAPITAL LETTER of the correct answer on the answer sheet provided.

1. A summer camp offers canoeing (C), rock climbing (R), and archery (A). The following Venn diagram shows the types of activities the campers like. Determine $n((A \cup R) \backslash C)$.
(A) 8
(B) 13
(C) 34
(D) 53

2. Which represents the shaded area in the Venn diagram below?

(A) $\quad A \backslash B(A$ minus $B)$
(B) $A \cup B$ (A or B)
(C) $\quad B \backslash A(B$ minus $A)$
(D) $\quad B^{\prime}(\operatorname{not} B)$
3. There are 42 guests at a golf resort. Of these guests, 25 plan to go golfing (G) and 22 plan to play beach volleyball $(\mathrm{V})$. There are 10 guests who do not plan to golf or play beach volleyball. How many guests plan to golf and play beach volleyball, $n(G \cap V)$ ?
(A) 15
(B) 22
(C) 37
(D) 42
4. A is the set of positive odd integers less than 11. B is the set of multiples of 3 between 2 and 17. How many elements are not in the intersection of A and $\mathrm{B}, n\left((A \cap B)^{\prime}\right)$.
(A) 1
(B) 2
(C) 6
(D) 7
5. Which shows the total number of possible arrangements of 8 distinguishable objects taken 5 at a time?
(A) ${ }_{5} C_{8}$
(B) ${ }_{8} C_{5}$
(C) $\quad{ }_{5} P_{8}$
(D) ${ }_{8} P_{5}$
6. In how many ways can the letters in the word CART be arranged?
(A) 1
(B) 4
(C) 12
(D) 24
7. Which is a non-permissible value of n in the rational expression $\frac{(n+1)!}{(n+2)!}$ ?
(A) -2
(B) -1
(C) 0
(D) 1
8. Simplify: $\frac{(n-3)!}{(n-1)!}$
(A) $\frac{1}{n^{2}-n}$
(B) $\frac{1}{n^{2}-3 n+2}$
(C) $n^{2}-n$
(D) $n^{2}-3 n+2$
9. There are 8 marbles in a bag: 2 red, 3 white, and 3 blue. If taken out one at a time, in how many different ways can all 8 marbles be taken out of the bag?
(A) 280
(B) 560
(C) 6720
(D) 40320
10. To get from his home to school, John must travel by only heading East (E) or South (S). One example of a route is given representing four moves East followed by three moves South (EEEESSS). Under these rules, which represents the total number of possible routes that can be taken to get from John's house to his school?
(A) $\frac{7!}{4!3!}$
(B) $\frac{12!}{4!3!}$
(C) 7 !
(D) 12 !
11. If the odds against the Toronto Maple Leafs winning their next game is 3:7, what is the probability that they will win their next game?
(A) $\frac{3}{10}$
(B) $\frac{3}{7}$
(C) $\frac{7}{10}$
(D) $\frac{7}{3}$
12. A committee of three people will be randomly chosen from a group of eight people; 5 females and 3 males. Which represents the probability of selecting a committee that has at least one male and at least one female member?
(A)

$$
\frac{\left({ }_{3} C_{1} \times{ }_{5} C_{2}\right)+\left({ }_{3} C_{2} \times{ }_{5} C_{1}\right)}{{ }_{8} C_{3}}
$$

(B) $\frac{\left({ }_{3} C_{0} \times{ }_{5} C_{3}\right)+\left({ }_{3} C_{3} \times{ }_{5} C_{0}\right)}{{ }_{8} C_{3}}$
(C) $\frac{\left({ }_{8} C_{1} \times{ }_{5} C_{2}\right)+\left({ }_{8} C_{2} \times{ }_{5} C_{1}\right)}{\left({ }_{3} C_{3} \times{ }_{5} C_{3}\right)}$
(D) $\frac{\left({ }_{8} C_{1} \times{ }_{3} C_{2}\right)+\left({ }_{8} C_{2} \times{ }_{3} C_{1}\right)}{\left({ }_{3} C_{3} \times{ }_{5} C_{3}\right)}$
13. Shavana estimates that her probability of passing French is 0.7 and her probability of passing chemistry is 0.6 . What is the probability that Shavana will fail French, but pass chemistry?
(A) $12 \%$
(B) $18 \%$
(C) $28 \%$
(D) $42 \%$
14. There are 35 athletes in Charlie's Grade 12 class. Based on a survey he conducted, he knows that 27 of these students play hockey and 10 play golf. What is the probability that a student chosen at random will play golf only?
(A) $\frac{2}{35}$
(B) $\frac{8}{35}$
(C) $\frac{2}{7}$
(D) $\frac{5}{7}$
15. Students were asked to find the probability of choosing an ace from a standard deck of 52 cards and then choosing a king from the same deck of cards without replacement. Which student has the correct solution?


| Student | Solution |
| :---: | :---: |
| Abby | $\left(\frac{1}{13}\right)\left(\frac{1}{13}\right)$ |
| Devon | $\left(\frac{1}{13}\right)+\left(\frac{1}{13}\right)$ |
| Robyn | $\left(\frac{1}{13}\right)\left(\frac{4}{51}\right)$ |
| Ron | $\left(\frac{1}{13}\right)+\left(\frac{4}{51}\right)$ |

(A) Abby
(B) Devon
(C) Robyn
(D) Ron
16. What are the non-permissible value(s) of $x$ in the rational expression $\frac{x+2}{3(x-2)(x-4)}$ ?
(A) $-2,2,4$
(B) 0
(C) 2, 3, 4
(D) 2,4
17. What is the simplified form of $\frac{x^{3}}{x^{3}-6 x^{2}}, x \neq 0,6$ ?
(A) $-6 x^{2}$
(B) $-\frac{1}{6 x^{2}}$
(C) $\frac{x}{x-6}$
(D) $\frac{x-6}{x-6}$
18. What expression is equivalent to $\frac{x-1}{x+1}, x \neq 1$ ?
(A) $\frac{x^{2}-x}{x^{2}+x}$
(B) $\frac{2 x-2}{x+1}$
(C) $\frac{2 x-1}{2 x+1}$
(D) $\frac{2 x-2}{2 x+2}$
19. Simplify: $\frac{3 x-9}{6 x^{2}-54}$
(A) $\frac{-1}{2 x-6}, x \neq-3,3$
(B) $\frac{-1}{2 x+6}, x \neq-3,3$
(C) $\frac{1}{2 x+6}, x \neq-3,3$
(D) $\frac{1}{2 x-6}, x \neq-3,3$
20. Simplify: $\frac{8}{3 x} \div \frac{4 x-4}{6 x^{2}}$
(A) $\frac{16 x-16}{9 x^{3}}, x \neq 0,1$
(B) $\frac{4 x}{x-1}, x \neq 0,1$
(C) $\frac{x-1}{4 x}, x \neq 0,1$
(D) $\frac{9 x^{3}}{16 x-16}, x \neq 0,1$
21. Simplify: $\frac{2 x}{x+3}-\frac{5 x}{2 x+6}$
(A) $\frac{-3 x}{-x-9}, x \neq-3$
(B) $\frac{-3 x}{-x-3}, x \neq-3$
(C) $\frac{-x}{2(x+3)}, x \neq-3$
(D) $\frac{x}{2(x+3)}, x \neq-3$
22. How many turning points are on the graph of the function $f(x)=2 x^{2}-x+7$ ?
(A) 0
(B) 1
(C) 2
(D) 3
23. What is the y-intercept of the equation $f(x)=2 x^{2}+7 x-8$ ?
(A) -8
(B) 0
(C) 7
(D) 8
24. Which graph represents a function with the characteristics listed below?

- Two x-intercepts
- Extending from Quadrant III to Quadrant I
(A)

(B)

(D)

25. 

(A)


Which graph match the equation $f(x)=-3 x^{2}-4$ ?


(B)

(C)

(D)

26. Determine the sign of the leading coefficient and the value of the constant term in the graph below.
(A) negative and 0
(B) negative and 4
(C) positive and 0
(D) positive and 4

27. From which quadrants does the graph of $f(x)=-x^{3}+3 x^{2}-4$ extend?
(A) II to I
(B) II to IV
(C) III to I
(D) III to IV
28. Which is an increasing exponential function?
(A) $f(x)=\frac{1}{3}\left(\frac{5}{2}\right)^{x}$
(B) $\quad f(x)=0.5(0.5)^{x}$
(C) $\quad f(x)=\frac{3}{2}(1)^{x}$
(D) $f(x)=2\left(\frac{3}{4}\right)^{x}$
29. Which equation describes a graph with a y-intercept of 4 that doubles in amount every 5 years?
(A) $y=4\left(\frac{1}{2}\right)^{\frac{t}{5}}$
(B) $y=5\left(\frac{1}{2}\right)^{\frac{t}{4}}$
(C) $y=4(2)^{\frac{t}{5}}$
(D) $y=5(2)^{\frac{t}{4}}$
30. The population of a strain of bacteria growing in a Petri dish is modeled by the function $P(t)=6000(2)^{\frac{t}{7}}$, where $P(t)$ represents the number of bacteria and $t$ represents the time in hours after the initial count. How much time will it take for the number of bacteria to reach 24000 ?
(A) 7 h
(B) 14 h
(C) 28 h
(D) 56 h
31. Solve for $\mathrm{x}: \quad 3^{7 x+1}=27^{x-1}$
(A) -1
(B) $-\frac{1}{2}$
(C) $\frac{1}{2}$
(D) 1
32. What is the equation that represents a $\$ 5000$ loan with an annual interest rate of $4 \%$ that is compounded quarterly?
(A) $\quad A=0.01(5000)^{n}$
(B) $\quad A=0.04(5000)^{n}$
(C) $\quad A=5000(1.01)^{n}$
(D) $\quad A=5000(1.04)^{n}$
33. Which is true of the table given below?

| $\mathbf{x}$ (years) | 0 | 3 | 6 | 9 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{y}$ (amount) | 320 | 160 | 80 | 40 | 20 |

Initial Amount
(A) 160
(B) 160
(C) 320
(D) 320

Amount Growth
reduces to half every three years reduces to one third every two years reduces to half every three years reduces to one third every two years
34. Which graph represents $y=-2 \ln x$ ?
(A)

(B)

(C)

(D)

35. What is $3 \log _{5} 3-\log _{5} 9$ written as a single logarithm?
(A) $\quad \log _{5} 1$
(B) $\quad \log _{5} 3$
(C) $\quad \log _{5} 81$
(D) $\quad \log _{5} 243$
36. Evaluate: $\log _{\frac{1}{3}} 81$
(A) -27
(B) $\quad-4$
(C) 4
(D) 27
37. What is the exponential form of $d=\log _{C} 5$ ?
(A) $\quad C=5^{d}$
(B) $5=C^{d}$
(C) $d=5^{c}$
(D) $5=d^{C}$
38. Solve for $\mathrm{x}: \quad 3^{x-1}=8$
(A) $\frac{\log 3}{\log 8}+1$
(B) $\frac{\log 3+1}{\log 8}$
(C) $\frac{\log 8}{\log 3}+1$
(D) $\frac{\log 8+1}{\log 3}$
39. The equation $A(t)=A_{0}\left(\frac{1}{2}\right)^{\frac{t}{5}}$ represents a radioactive sample after $t$ years. How much time will it take for $70 \%$ of the sample to decay?
(A) 2.6 years
(B) 3.0 years
(C) 7.0 years
(D) 8.7 years
40. Which graph represents a function that is periodic and sinusoidal?
(A)

(B)

(C)

(D)

41. What is the amplitude and maximum value for the function $f(x)=3 \sin 2\left(x-90^{\circ}\right)+1$ ?

|  | Amplitude | Maximum Value |
| :---: | :---: | :---: |
| (A) | 2 | 3 |
| (B) | 2 | 4 |
| (C) | 3 | 3 |
| (D) | 3 | 4 |

42. The graph of which function has a period of $90^{\circ}$ ?
(A) $y=3 \cos \frac{1}{4} x-1$
(B) $y=3 \cos \left(x-90^{\circ}\right)-1$
(C) $y=4 \cos \left(x+90^{\circ}\right)+1$
(D) $y=4 \cos 4 x+1$
43. What is the value of $\frac{5 \pi}{6}$ in degrees?
(A) $150^{\circ}$
(B) $170^{\circ}$
(C) $216^{\circ}$
(D) $300^{\circ}$
44. What is the range of the function $y=8 \cos x-4$ ?
(A) $\{x \mid-12 \leq x \leq 4, x \in R\}$
(B) $\{x \in R\}$
(C) $\{y \mid-12 \leq y \leq 4, y \in R\}$
(D) $\{y \in R\}$
45. The graph of the function $y=4 \cos 3 x$ has its amplitude halved and its period doubled. Which represents the new function?
(A) $y=2 \cos \frac{3}{2} x$
(B) $y=2 \cos 6 x$
(C) $y=8 \cos \frac{3}{2} x$
(D) $y=8 \cos 6 x$
46. The interest rate on the loan shown in the chart below is $5 \%$ compounded monthly. How much of the third payment is interest?

| Payment <br> Period <br> (month) | Payment <br> $(\$)$ | Principal <br> Paid (\$) | Balance (\$) |
| :---: | :---: | :---: | :---: |
| 0 |  |  | 15,000 |
| 1 | 450 | 387.50 | $14,612.50$ |
| 2 | 450 | 389.11 | $14,223.39$ |
| 3 | 450 | 390.74 | $13,832.65$ |

(A) $\$ 59.26$
(B) $\$ 60.89$
(C) $\$ 62.50$
(D) $\$ 182.65$
47. 24 semi-annual payments are required to pay off a loan. How many years does this represent?
(A) 1
(B) 2
(C) 12
(D) 24
48. The equation $A=1000(1.005)^{36}$ represents a bank loan that is compounded monthly. How long has interest accumulated on the loan?
(A) 1.5 years
(B) 3 years
(C) 5 years
(D) 36 years
49. Which represents the highest interest that would be paid?

|  | Interest rate | Compounded |
| :---: | :---: | :---: |
| (A) | $12 \%$ | daily |
| (B) | $12 \%$ | monthly |
| (C) | $19 \%$ | daily |
| (D) | $19 \%$ | monthly |

50. A student repaid a total of $\$ 2536.32$ for a loan including the principal and interest. The interest rate was $8 \%$ compounded quarterly for 2 years. What was the principal amount of the loan to the nearest cent?
(A) $\$ 1864.27$
(B) $\$ 2164.72$
(C) $\$ 2174.48$
(D) $\$ 2437.83$

PART II
Total Value: 50 marks
Answer ALL items in the space provided. Show ALL workings.
Value
3 51. 40 members in a sports club were surveyed:
23 play ball hockey
24 play tennis
18 play golf
14 play tennis and ball hockey
8 play tennis and golf
2 play all three sports
1 member makes the refreshments and does not play any sport
Determine the number of people who play ball hockey only.

52.(a) In Newfoundland and Labrador a license plate consists of a letter-letter-letter-digit-digit-digit arrangement such as CXT 132. How many license plate arrangements are possible if no letter can be repeated but numbers can? (There are 26 letters in the alphabet)
52.(b) How many different arrangements of the letters in the word MISSISSIPPI are possible?
52.(c) A committee of six (6) members must be chosen from a group of four (4) males and seven (7) females. In how many ways can the committee be chosen if there must be three (3) males and three (3) females on it?
53.(a) When considering the total arrangements of the letters in the word MOUTH, what is the probability of the vowels $(\mathrm{O}, \mathrm{U})$ being together?
53.(b) John is sometimes late for school in the mornings. Based on his track record, there is an $85 \%$ chance of being on time when he remembers to set his alarm clock and a $40 \%$ chance when he doesn't. Tomorrow there is a $60 \%$ chance that John will set his alarm clock. What is the probability that he will be on time for school tomorrow?

$$
4 \text { 54.(a) Simplify and state any restrictions: } \frac{9-x^{2}}{x^{2}} \div \frac{x+3}{3 x}
$$

2 54.(b) Tyrone can mow a lawn in 3 hours. When Tyrone and Devon mow the same lawn together they can finish it in $\frac{12}{7}$ hours. How long would it take Devon to mow this lawn on his own?

3 55.(a) A spherical balloon is being inflated. The surface area (SA), in square meters, is related to the time in minutes as show in the table below.

| Time $(s)$ | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| SA $\left(m^{2}\right)$ | 13 | 28 | 50 | 79 | 113 |

Using a graphing calculator, the quadratic regression function give as follows:
QuadReg:
$y=a x^{2}+b x+c$
$a=3.214$
$b=12.243$
$c=12.829$
Write the quadratic function that best models this data and then use the function to find the time at which the balloon has a surface area of $40 \mathrm{~m}^{2}$. Be sure to include what each variable represents.
55.(b) Given the function $f(x)=-4 x^{3}+2 x^{2}-3 x+56$, complete the table to describe its characteristics.
i)

| $y$-intercept |  |
| :---: | :--- |
| end behaviour <br> (left and right) |  |
| Max \# of possible <br> x-intercepts |  |

ii) Explain why the graph of this function has the range $\{y \in R\}$.

4 56.(a) Algebraically solve for $\mathrm{x}: \quad \sqrt{\frac{1}{2}}=4^{5 x+1}$
56.(b) The amount of a substance present after $x$ hours is given in the table below.

| Time (hours) | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Amount $(\mathrm{g})$ | 2000 | 1000 | 500 | 250 | 125 |

(i) Use this information to create an equation of the form $y=a b^{x}$ that models the data.
(ii) Use this equation to find the time at which the amount present is 31.25 g .
interest will Sara be charged for borrowing the money?

## Selected Response

Answer Sheet

1. $\qquad$
2. $\qquad$
3. $\qquad$
4. $\qquad$
5. $\qquad$
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48. $\qquad$
49. $\qquad$
50. $\qquad$
