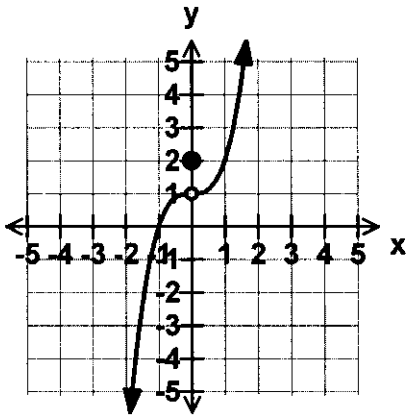
Part I – Selected Response (20 marks) *– Circle the letter of the response that best completes the statement or answers the question.*

1. What is the value of $\lim_{x \to 0} f(x)$ for the graph below?

B

A.) -1
B.) 1
C.) 2
D.) Does not exist



2. Evaluate: $\lim_{x \to 0} \left(\frac{x^4 - x^3 - 4x}{x} \right)$

A

A.) -4 $\lim_{x \to 0} \frac{x(x^3 - x^2 - 4)}{x}$ B.) -1
C.) 1 D.) Undefined

3. If f and g are continuous functions with $f(9) = 2$ and $\lim_{x \to 9} [2f(x) - g(x)] = 9$, find $g(9)$.

A

A.) -5 B.) 4
C.) 11 D.) 13

4. Find the limit $\lim_{x \to 3} \frac{x^2 - x - 6}{x - 3}$, if it exists.

C

A.) 2 $\frac{(x-3)(x+2)}{x-3}$ B.) 3
C.) 5 D.) Does not exist

5. Determine where f is discontinuous.

$$f(x) = \begin{cases} \sqrt{-x}, & x < 0 \\ 11 - x, & 0 \le x < 11 \\ (11-x)^2, & x > 11 \end{cases}$$

A.) -11 and 0
B.) 0 only
C.) 0 and 11
D.) 11 only

C

6. What is the horizontal asymptote of the limit $\lim_{x \to \infty} \frac{2x+1}{3x^2+7}$?

B

A.) ⅔ B.) 0
C.) ½ D.) None

7. What type of discontinuity is represented by the function $f(x) = \frac{x^2-4}{x+2}$?

$\frac{(x+2)(x-2)}{x+2}$

A.) Infinite B.) Jump
C.) Removable D.) Oscillating

C

8. What is the point of discontinuity for $f(x) = \frac{x^2-1}{x+1}$?

$\frac{(x+1)(x-1)}{x+1}$

A.) (-1, 2) B.) (1, 2)
C.) (-1, -2) D.) (1, -2)

C

(-1, -2)

9. For what value of the constant c is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx + 7, & x \le 2 \\ cx^2 - 5, & x > 2 \end{cases}$$

D

$c(2) + 7 = c(2)^2 - 5$

$2c + 7 = 4c - 5$

$-2c = -12$

$c = 6$

A.) -6 B.) -2

C.) 2 D.) 6

10. What is the oblique asymptote for the $\lim_{x \to \infty} \frac{2x^3 + 3x^2 + x}{x^2 - x + 1}$?

C

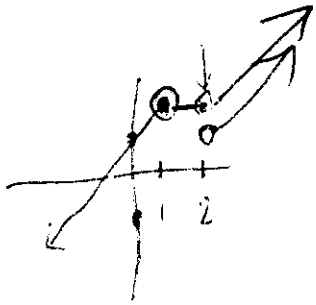
A.) 2x B.) 2x + 1

C.) 2x + 5 D.) 2x - 3

Part II – Constructed Response (30 marks) *– Complete all questions in the space provided. Give complete solutions using the appropriate notation and justification.*

11. A student in Math 1000 at MUN says that the following piece-wise function is ***discontinuous at two different values of x***. Do you agree or disagree. **(3 marks)**

$$f(x) = \begin{cases} x + 1, & x < 1 \\ 2, & 1 \le x \le 2 \\ x - 1, & x > 2 \end{cases}$$



Disagree

$x = 1$

$\lim_{x \to 1^-} f(x) = 2$

$\lim_{x \to 1^+} f(x) = 2$

$f(1) = 2$

∴ Continuous

$x = 2$

$\lim_{x \to 2^-} f(x) = 2$

$\lim_{x \to 2^+} f(x) \neq$

$f(2) = 2$

discontinuous

12. Use the graph of $y = f(x)$, shown below, to evaluate each of the following. Assign a value of $+\infty$ or $-\infty$, where appropriate. (***No workings are required here!***) **(4 marks)**

(a) $\lim_{x \to -4} f(x) =$ 1

(b) $\lim_{x \to -2^+} f(x) =$ 0

(c) $\lim_{x \to 1} f(x) =$ does not exist

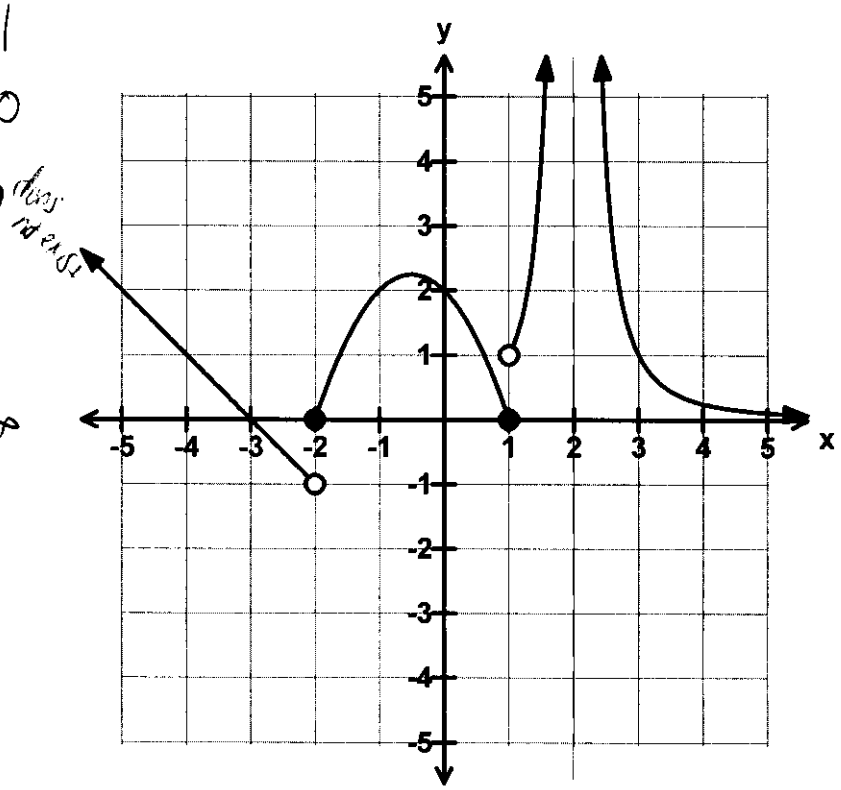
(d) $f(2) =$ $+\infty$

(e) $\lim_{x \to 0} f(x) =$ 2

(f) $\lim_{x \to -\infty} f(x) = +\infty$

(g) $\lim_{x \to 2} f(x) = +\infty$

(h) $\lim_{x \to \infty} f(x) =$ 0



13. Evaluate using the ***laws of limits***. **(2 marks)**

$$\lim_{x \to -2} \frac{x^2 - 2x - 8}{x^2 + 5x + 6}$$

$$\lim_{x \to -2} x^2 - 2\lim_{x \to -2} x - \lim_{x \to -2} 6$$

$$= \frac{\lim_{x \to -2} x^2 - 2\lim_{x \to -2} x - \lim_{x \to -2} 6}{\lim_{x \to -2} x^2 + 5\lim_{x \to -2} x + \lim_{x \to -2} 6}$$

$$= \frac{(-2)^2 - 2(-2) - 6}{(-2)^2 + 5(-2) + 6} = \frac{4 + 4 - 6}{4 - 10 + 6} = \frac{0}{0}$$

$$\frac{(x-4)(x+2)}{(x+2)(x+3)}$$

$$\frac{x-4}{x+3}$$

$$\frac{-6}{1} = -6$$

14. Evaluate each of the following limits, if it exists. If it fails to exist, justify your answer. **(21 marks)**

a.) $\lim_{x \to -1} \frac{x+1}{x^2-x-2}$

$\lim_{x \to -1} \frac{x+1}{(x+1)(x-2)}$

$\lim_{x \to -1} \frac{1}{x-2}$

$= \frac{-1}{3}$

b.) $\lim_{x \to 0} \frac{(x+1)^3 - 1}{x}$

$\lim_{x \to 0} \frac{(x^3 + 2x^3 + 3x + 1) - 1}{x}$

$\lim_{x \to 0} \frac{x^3 + 2x^3 + 3x}{x}$

$\lim_{x \to 0} \frac{x(x^2 + 2x + 3)}{}$

$\lim_{x \to 0} x^2 + 2x + 3$

$= 0 + 0 + 3$

$= 3$

c.) $\lim_{x \to \infty} \left(\frac{3x}{x+2} - \frac{x-1}{2x+6} \right)$

$\lim_{x \to \infty} \left(\frac{3x}{x+2} - \frac{x-1}{2(x+3)} \right)$

$\lim_{x \to \infty} \frac{3x(2(x+3)) - (x-1)(x+2)}{2(x+2)(x+3)}$

$\lim_{x \to \infty} \frac{6x^2 + 18x - (x^2 + x - 2)}{2(x+2)(x+3)}$

$\lim_{x \to \infty} \frac{6x^2 + 18x - x^2 - x + 2}{2(x^2 + 5x + 6)}$

$\lim_{x \to \infty} \frac{5x^2 + 17x + 2}{2x^2 + 10x + 12}$

$= \frac{5}{2}$

d.) $\lim_{x \to 3} \frac{x^2 + 2x - 15}{2x^2 - 5x - 3}$

$\lim_{x \to 3} \frac{(x+5)(x-3)}{(2x+1)(x-3)}$

$\lim_{x \to 3} \frac{x+5}{2x+1}$

$\frac{3+5}{2(3)+1}$

$\frac{8}{7}$