

**MATH 3208  
TEST # 2**

**UNIT 1: FUNCTIONS/LIMITS/CONTINUITY**

**NAME:** \_\_\_\_\_

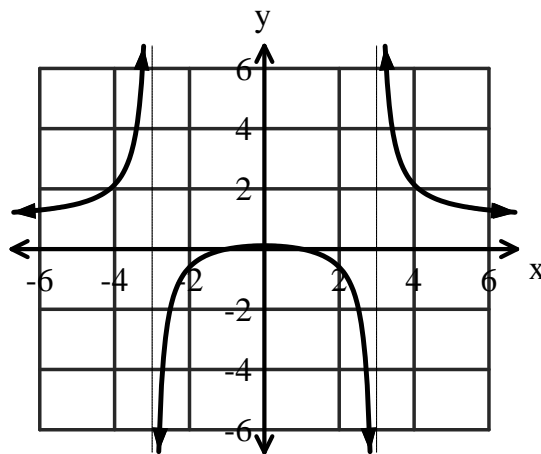
**PART A: MULTIPLE CHOICE ( Value: 20 )**

Circle the letter of the correct response.

1. What is the domain of the function  $f(x) = \frac{x+3}{x^2-3x}$  ?

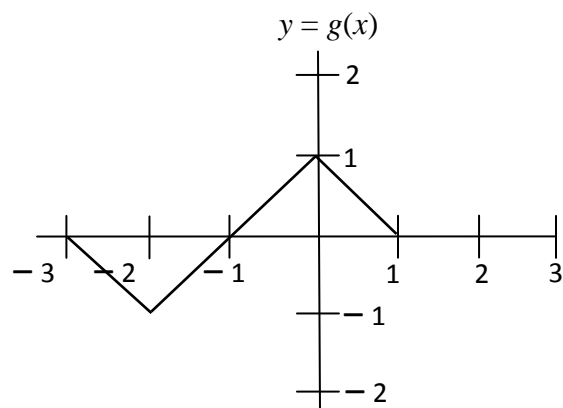
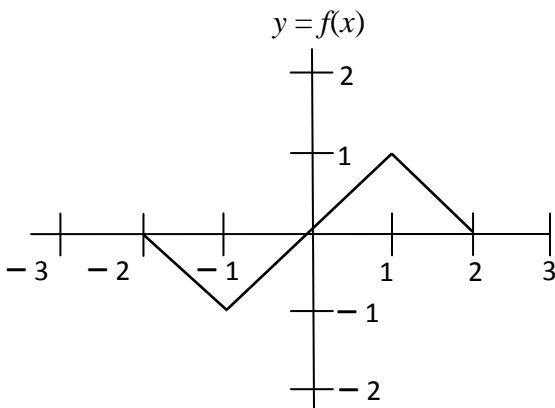
- (A)  $x \in (-\infty, -3) \cup (0, 3) \cup (3, \infty)$       (B)  $x \in (-\infty, 0) \cup (0, 3) \cup (3, \infty)$   
 (C)  $x \in (-\infty, -3) \cup (-3, 0) \cup (0, \infty)$       (D)  $x \in (-\infty, \infty)$

2. What are the intervals of decrease for the function graphed below?



- (A)  $x \in (-\infty, -3) \cup (-3, 0)$       (B)  $x \in (-\infty, -3) \cup (0, 3)$   
 (C)  $x \in (-3, 0) \cup (3, \infty)$       (D)  $x \in (0, 3) \cup (3, \infty)$

3. Given the graphs of  $y = f(x)$  and  $y = g(x)$ . Which function is represented by  $g(x)$ ?



- (A)  $g(x) = f(x-1)$       (B)  $g(x) = f(x+1)$       (C)  $g(x) = f(x) - 1$       (D)  $g(x) = f(x) + 1$

4. Evaluate the limit:  $\lim_{x \rightarrow 0} \frac{(x+1)^2 - 1}{x}$

- (A) 0      (B) 1      (C) 2      (D) does not exist

5. Evaluate the limit:  $\lim_{x \rightarrow 2^+} \frac{|x-2|}{x^2 - 4x + 4}$       (A)  $\frac{1}{4}$       (B)  $-\frac{1}{4}$       (C)  $\infty$       (D)  $-\infty$

6. Evaluate the limit:  $\lim_{x \rightarrow -2} \frac{x^3 + 8}{x^2 - 2x - 8}$

- (A)  $-2$       (B)  $2$       (C)  $6$       (D) does not exist

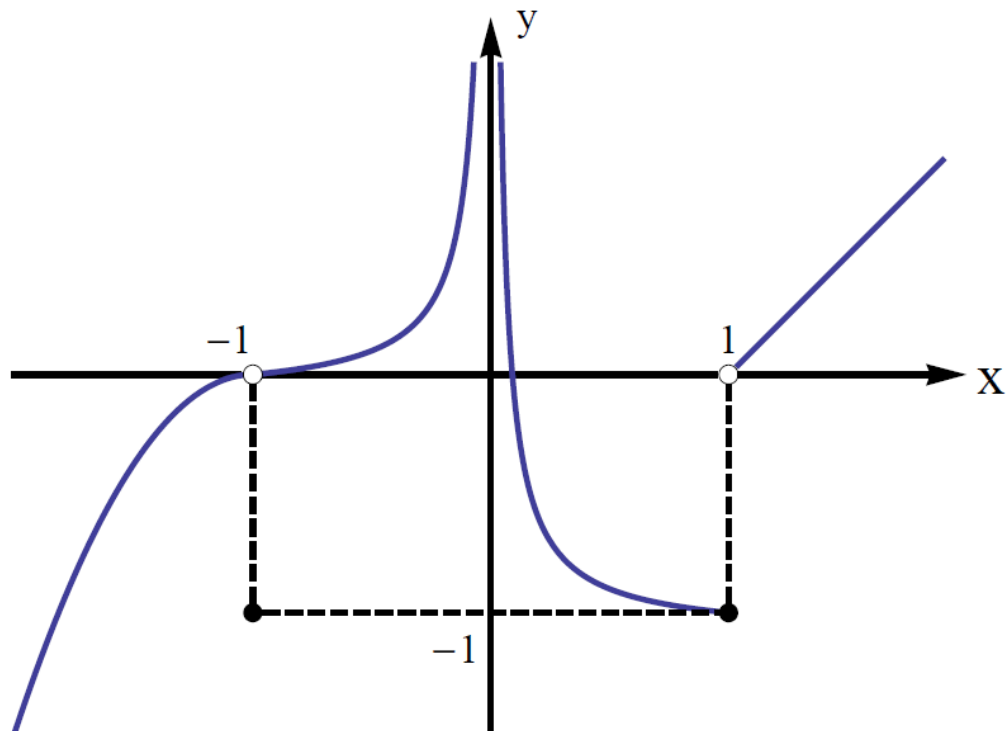
7. Evaluate the limit:  $\lim_{x \rightarrow 0} \frac{\frac{1}{x} + \frac{1}{3}}{\frac{2}{x}}$

- (A)  $0$       (B)  $\frac{1}{2}$       (C)  $\frac{1}{3}$       (D) does not exist

8. Evaluate the limit:  $\lim_{x \rightarrow 0} \frac{\tan 3x}{\sin 2x}$

- (A) does not exist      (B)  $\frac{1}{2}$       (C)  $\frac{2}{3}$       (D)  $\frac{3}{2}$

Use the given graph to answer questions # 9 – 12.



9. Determine the value of  $f(1)$ .

- (A)  $-1$       (B)  $0$       (C)  $1$       (D) undefined

10. Evaluate the limit:  $\lim_{x \rightarrow 0^-} f(x)$

- (A)  $-\infty$       (B)  $\infty$       (C)  $-1$       (D) does not exist

11. Evaluate the limit:  $\lim_{x \rightarrow -1} f(x)$

- (A)  $-1$       (B)  $1$       (C)  $0$       (D) does not exist

12. At which value does the graph of the function have a *jump discontinuity*?

- (A) none exist      (B)  $x = -1$       (C)  $x = 0$       (D)  $x = 1$

13. Given  $f(x) = \sqrt{\frac{x^2 - 25}{x^2 - 9}}$  on which interval(s) is the function  $f(x)$  continuous?

- (A)  $x \in (-\infty, -5] \cup (-3, 3) \cup [5, \infty)$       (B)  $x \in [-5, -3) \cup (3, 5]$   
 (C)  $x \in (-\infty, -5) \cup [-3, 3] \cup (5, \infty)$       (D)  $x \in (-5, -3] \cup [3, 5)$

14. Which function has a removable discontinuity at  $x = 1$ ?

- (A)  $f(x) = \frac{x^2 + 3x + 2}{x^2 - 1}$       (B)  $f(x) = \frac{x^2 - 3x + 2}{x^2 - 1}$   
 (C)  $f(x) = \frac{x^2 - 1}{x^2 + 3x + 2}$       (D)  $f(x) = \frac{x^2 - 1}{x^2 - x - 2}$

15. Which statement is **TRUE** for the piecewise function  $f(x) = \begin{cases} x^2 + 3x & \text{if } x < -3 \\ x^2 - 9 & \text{if } -3 \leq x \leq 1 \\ 3x - 2 & \text{if } x > 1 \end{cases}$ ?

- (A) The function is right-continuous at  $x = 1$ .      (B) The function is right-continuous at  $x = -3$ .  
 (C) The function is left-continuous at  $x = -3$ .      (D) The function is left-continuous at  $x = 1$ .

16. Which statement is **FALSE** for the function  $f(x) = \frac{2x^2 - 7x + 3}{x^2 - 2x - 3}$ ?

- (A) The point of discontinuity is  $\left(3, \frac{5}{4}\right)$ .      (B) The horizontal asymptote is  $x = 2$ .  
 (C) The  $x$ -intercept is  $\frac{1}{2}$ .      (D) The vertical asymptote is  $x = 1$ .

17. What is the oblique asymptote of the function  $f(x) = \frac{x^2 + 5x + 2}{x - 1}$ ?

- (A)  $y = x - 4$       (B)  $y = x + 4$       (C)  $y = x - 6$       (D)  $y = x + 6$

18. Evaluate the limit:  $\lim_{x \rightarrow -\infty} \frac{x^3}{2 + x^2}$

- (A) 0      (B) -2      (C)  $-\infty$       (D)  $\infty$

Use the given function to answer questions # 19 – 20.

$$f(x) = \begin{cases} k^2 - xk & \text{if } x < 2 \\ k^2 + 6 & \text{if } x = 2 \\ k^2x + k & \text{if } x > 2 \end{cases}$$

19. Determine the values of  $k$  for which the  $\lim_{x \rightarrow 2} f(x)$  exists?

- (A)  $k = 0, 3$       (B)  $k = 0, -3$       (C)  $k = 3, -3$       (D)  $k = 0, -1$

20. Determine the value of  $k$  for which  $f(x)$  is continuous at  $x = 2$ .

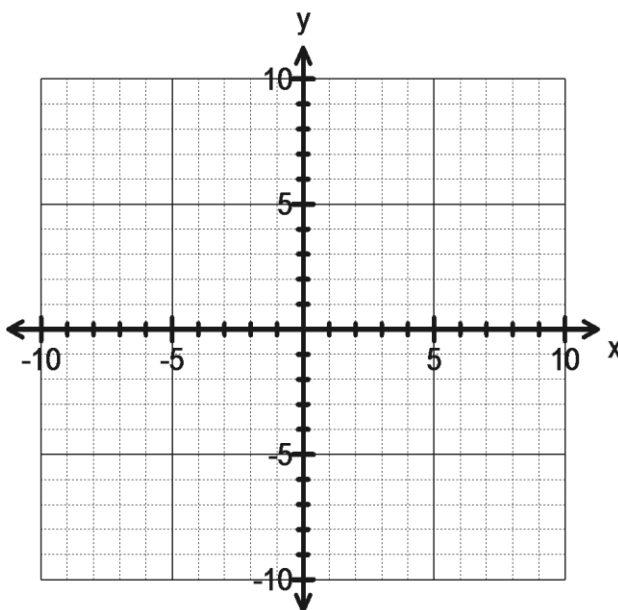
- (A)  $k = -3$       (B)  $k = 3$       (C)  $k = -1$       (D)  $k = 0$

**PART B: QUESTIONS ( Value: 32 )**

Answer all questions in the space provided. Show all workings to ensure full marks!

21. Sketch the graph of the given function and evaluate  $\lim_{x \rightarrow 0} f(x)$ . ( 5 )

$$f(x) = \frac{4x + |x|}{2x - |x|}$$



22. Evaluate the following limits. ( 12 )

$$(a) \lim_{x \rightarrow -1} \frac{x^2 + 4x + 3}{x^3 + x^2 - 9x - 9}$$

$$(b) \lim_{x \rightarrow 2} \frac{x^2 - 6x + 8}{\sqrt{x^2 + 5} - 3}$$

$$(c) \lim_{x \rightarrow 2} \frac{|6 - 3x|}{x^2 - x - 2}$$

23. Given the function  $f(x) = \begin{cases} \frac{x^2 + x - 2}{x - 5} & \text{if } x \geq -1 \\ \frac{2x + 6}{x^2 - 9} & \text{if } x < -1 \end{cases}$ , using the definition of continuity

determine all points at which  $f(x)$  is discontinuous. Classify any discontinuities as removable or non-removable. Show your limit workings to verify your answer. (7)

24. Determine the equation of all horizontal asymptotes and all vertical asymptotes of the following functions. For vertical asymptotes, determine the behavior of the function as it approaches the vertical asymptote from each side. (8)

(a)  $f(x) = \frac{2x^2 + 5x + 2}{x^2 - x - 6}$

(b)  $f(x) = \frac{\sqrt{4x^2 + x}}{x + 1}$