Mathematics

Applied Mathematics 3202

Interim Edition

Newfoundland Labrador

Curriculum Guide
2013

Contents

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INTRODUCTION

Background

The curriculum guide communicates high expectations for students.

The Mathematics curriculum guides for Newfoundland and Labrador have been derived from *The Common Curriculum Framework for 10-12 Mathematics: Western and Northern Canadian Protocol*, January 2008. These guides incorporate the conceptual framework for Grades 10 to 12 Mathematics and the general outcomes, specific outcomes and achievement indicators established in the common curriculum framework. They also include suggestions for teaching and learning, suggested assessment strategies, and an identification of the associated resource match between the curriculum and authorized, as well as recommended, resource materials.

Beliefs About Students and Mathematics

Mathematical understanding is fostered when students build on their own experiences and prior knowledge.

Students are curious, active learners with individual interests, abilities and needs. They come to classrooms with varying knowledge, life experiences and backgrounds. A key component in developing mathematical literacy is making connections to these backgrounds and experiences.

Students learn by attaching meaning to what they do, and they need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. Through the use of manipulatives and a variety of pedagogical approaches, teachers can address the diverse learning styles, cultural backgrounds and developmental stages of students, and enhance within them the formation of sound, transferable mathematical understandings. Students at all levels benefit from working with a variety of materials, tools and contexts when constructing meaning about new mathematical ideas. Meaningful student discussions provide essential links among concrete, pictorial and symbolic representations of mathematical concepts.

The learning environment should value and respect the diversity of students' experiences and ways of thinking, so that students feel comfortable taking intellectual risks, asking questions and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. They must come to understand that it is acceptable to solve problems in a variety of ways and that a variety of solutions may be acceptable.

Affective Domain

To experience success, students must learn to set achievable goals and assess themselves as they work toward these goals.

A positive attitude is an important aspect of the affective domain and has a profound impact on learning. Environments that create a sense of belonging, encourage risk taking and provide opportunities for success help develop and maintain positive attitudes and self-confidence within students. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations and engage in reflective practices.

Teachers, students and parents need to recognize the relationship between the affective and cognitive domains, and attempt to nurture those aspects of the affective domain that contribute to positive attitudes. To experience success, students must learn to set achievable goals and assess themselves as they work toward these goals.

Striving toward success and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting, asssessing and revising personal goals.

Goals For Students

Mathematics education must prepare students to use mathematics confidently to solve problems.

The main goals of mathematics education are to prepare students to:

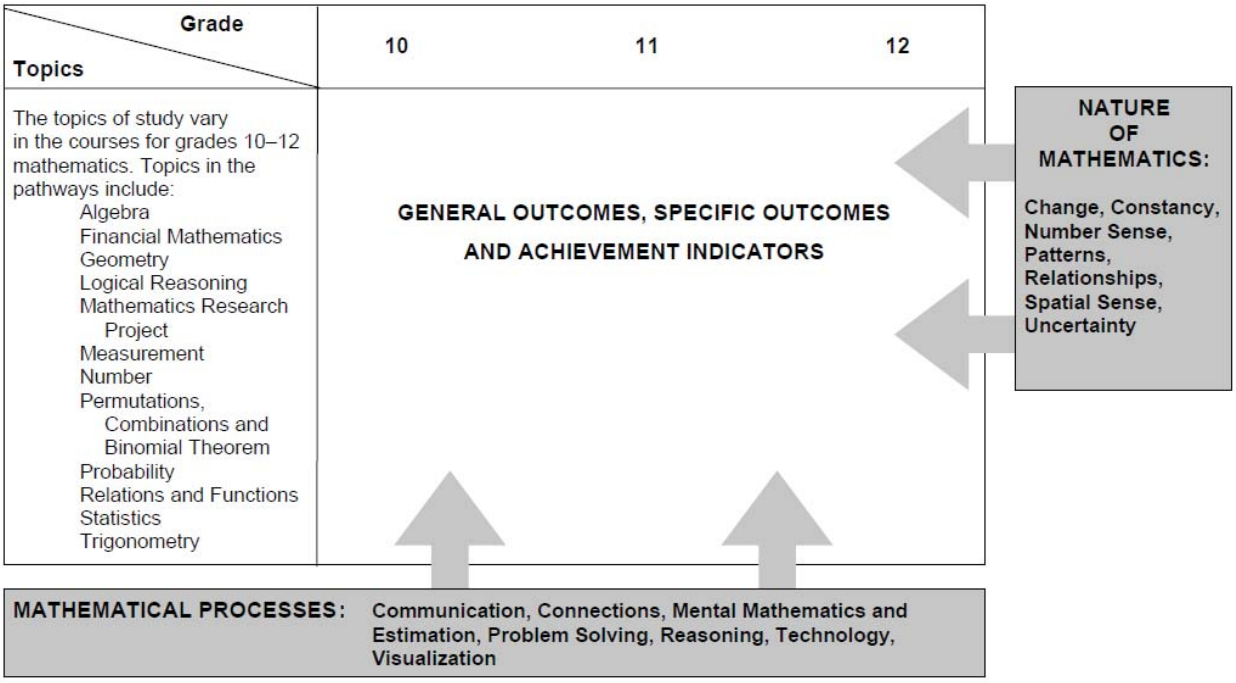
- use mathematics confidently to solve problems
- communicate and reason mathematically
- appreciate and value mathematics
- make connections between mathematics and its applications
- commit themselves to lifelong learning
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will:

- gain understanding and appreciation of the contributions of mathematics as a science, philosophy and art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity.

CONCEPTUAL FRAMEWORK FOR 10-12 MATHEMATICS

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



Mathematical Processes

- *Communication [C]*
- *Connections [CN]*
- *Mental Mathematics and Estimation [ME]*
- *Problem Solving [PS]*
- *Reasoning [R]*
- *Technology [T]*
- *Visualization [V]*

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and embrace lifelong learning in mathematics.
Students are expected to:

- communicate in order to learn and express their understanding
- connect mathematical ideas to other concepts in mathematics, to everyday experiences and to other disciplines
- demonstrate fluency with mental mathematics and estimation
- develop and apply new mathematical knowledge through problem solving
- develop mathematical reasoning
- select and use technologies as tools for learning and for solving problems
- develop visualization skills to assist in processing information, making connections and solving problems.

This curriculum guide incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning.

Communication [C]

Students must be able to communicate mathematical ideas in a variety of ways and contexts.

Students need opportunities to read about, represent, view, write about, listen to and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics.

Communication is important in clarifying, reinforcing and modifying ideas, attitudes and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology.

Communication helps students make connections among concrete, pictorial, symbolic, oral, written and mental representations of mathematical ideas.

Connections [CN]

Through connections, students begin to view mathematics as useful and relevant.

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students begin to view mathematics as useful, relevant and integrated.

Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged.

The brain is constantly looking for and making connections. "Because the learner is constantly searching for connections on many levels, educators need to *orchestrate the experiences* from which learners extract understanding … Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching" (Caine and Caine, 1991, p.5).

Mental Mathematics and Estimation [ME]

Mental mathematics and estimation are fundamental components of number sense.

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external memory aids.

Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy and flexibility.

"Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental math" (National Council of Teachers of Mathematics, May 2005).

Students proficient with mental mathematics "... become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving" (Rubenstein, 2001, p. 442).

Mental mathematics "... provides the cornerstone for all estimation processes, offering a variety of alternative algorithms and nonstandard techniques for finding answers" (Hope, 1988, p. v).

Estimation is used for determining approximate values or quantities or for determining the reasonableness of calculated values. It often uses benchmarks or referents. Students need to know when to estimate, how to estimate and what strategy to use.

Estimation assists individuals in making mathematical judgements and in developing useful, efficient strategies for dealing with situations in daily life.

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels.

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, "How would you know?" or "How could you ...?", the problem-solving approach is being modelled. Students develop their own problem-solving strategies by listening to, discussing and trying different strategies.

A problem-solving activity requires students to determine a way to get from what is known to what is unknown. If students have already been given steps to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is a powerful teaching tool that fosters multiple, creative and innovative solutions. Creating an environment where students openly seek and engage in a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive mathematical risk takers.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics.

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics.

Mathematical experiences in and out of the classroom provide opportunities for students to develop their ability to reason. Students can explore and record results, analyze observations, make and test generalizations from patterns, and reach new conclusions by building upon what is already known or assumed to be true.

Reasoning skills allow students to use a logical process to analyze a problem, reach a conclusion and justify or defend that conclusion.

Technology [T]

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures and solve problems.

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Technology can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts
- develop personal procedures for mathematical operations
- create geometric patterns
- simulate situations
- develop number sense.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels.

Visualization [V]

Visualization "involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world" (Armstrong, 1993, p. 10). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them.

Visualization is fostered through the use of concrete materials, technology and a variety of visual representations.

Visual images and visual reasoning are important components of number, spatial and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure, when to estimate and which estimation strategies to use (Shaw and Cliatt, 1989).

Nature of Mathematics

- *Change*
- *Constancy*
- *Number Sense*
- *Patterns*
- *Relationships*
- *Spatial Sense*
- *Uncertainty*

Mathematics is one way of trying to understand, interpret and describe our world. There are a number of components that define the nature of mathematics and these are woven throughout this curiculum guide. The components are change, constancy, number sense, patterns, relationships, spatial sense and uncertainty.

Change

Change is an integral part of mathematics and the learning of mathematics.

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics.

Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as:

- the number of a specific colour of beads in each row of a beaded design
- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen, 1990, p. 184).

Constancy

Constancy is described by the terms stability, conservation, equilibrium, steady state and symmetry.

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state and symmetry (AAAS-Benchmarks, 1993, p.270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include the following:

- The ratio of the circumference of a teepee to its diameter is the same regardless of the length of the teepee poles.
- The sum of the interior angles of any triangle is 180°.
- The theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations or the angle sums of polygons.

Number Sense

An intuition about number is the most important foundation of numeracy.

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (British Columbia Ministry of Education, 2000, p.146).

A true sense of number goes well beyond the skills of simply counting, memorizing facts and the situational rote use of algorithms. Mastery of number facts is expected to be attained by students as they develop their number sense. This mastery allows for facility with more complex computations but should not be attained at the expense of an understanding of number.

Number sense develops when students connect numbers to their own real-life experiences and when students use benchmarks and referents. This results in students who are computationally fluent and flexible with numbers and who have intuition about numbers. The evolving number sense typically comes as a by product of learning rather than through direct instruction. It can be developed by providing rich mathematical tasks that allow students to make connections to their own expereinces and their previous learning.

Patterns

Mathematics is about recognizing, describing and working with numerical and non-numerical patterns.

Mathematics is about recognizing, describing and working with numerical and non-numerical patterns. Patterns exist in all strands of mathematics.

Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with, and understanding of, their environment.

Patterns may be represented in concrete, visual or symbolic form. Students should develop fluency in moving from one representation to another.

Students must learn to recognize, extend, create and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving routine and non-routine problems.

Learning to work with patterns in the early grades helps students develop algebraic thinking, which is foundational for working with more abstract mathematics.

Relationships

Mathematics is used to describe and explain relationships.

Mathematics is one way to describe interconnectedness in a holistic worldview. Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects and concepts. The search for possible relationships involves collecting and analyzing data and describing relationships visually, symbolically, orally or in written form.

Spatial Sense

Spatial sense offers a way to interpret and reflect on the physical environment.

Spatial sense involves visualization, mental imagery and spatial reasoning. These skills are central to the understanding of mathematics.

Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3-D objects and 2-D shapes and to interpret and reflect on the physical environment and its 3-D or 2-D representations.

Some problems involve attaching numerals and appropriate units (measurement) to dimensions of shapes and objects. Spatial sense allows students to make predictions about the results of changing these dimensions; e.g., doubling the length of the side of a square increases the area by a factor of four. Ultimately, spatial sense enables students to communicate about shapes and objects and to create their own representations.

Uncertainty

Uncertainty is an inherent part of making predictions.

In mathematics, interpretations of data and the predictions made from data may lack certainty.

Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty.

The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation.

Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Essential Graduation Learnings

Essential graduation learnings are statements describing the knowledge, skills and attitudes expected of all students who graduate from high school. Essential graduation learnings are cross-curricular in nature and comprise different areas of learning: *aesthetic expression, citizenship, communication, personal development, problem solving, technological competence* and *spiritual and moral development*.

Aesthetic Expression

Graduates will be able to respond with critical awareness to various forms of the arts and be able to express themselves through the arts.

Citizenship

Graduates will be able to assess social, cultural, economic and environmental interdependence in a local and global context.

Communication

Graduates will be able to use the listening, viewing, speaking, reading and writing modes of language(s) and mathematical and scientific concepts and symbols to think, learn and communicate effectively.

Personal Development

Graduates will be able to continue to learn and to pursue an active, healthy lifestyle.

Problem Solving

Graduates will be able to use the strategies and processes needed to solve a wide variety of problems, including those requiring language and mathematical and scientific concepts.

Technological Competence

Graduates will be able to use a variety of technologies, demonstrate an understanding of technological applications, and apply appropriate technologies for solving problems.

Spiritual and Moral Development

Graduates will be able to demonstrate an understanding and appreciation for the place of belief systems in shaping the development of moral values and ethical conduct.

See *Foundations for the Atlantic Canada Mathematics Curriculum*, pages 4-6.

The mathematics curriculum is designed to make a significant contribution towards students' meeting each of the essential graduation learnings (EGLs), with the communication, problem-solving and technological competence EGLs relating particularly well to the mathematical processes.

Outcomes and Achievement Indicators

The curriculum is stated in terms of general outcomes, specific outcomes and achievement indicators.

General Outcomes

General outcomes are overarching statements about what students are expected to learn in each course.

Specific Outcomes

Specific outcomes are statements that identify the specific skills, understanding and knowledge that students are required to attain by the end of a given course.
In the specific outcomes, the word *including* indicates that any ensuing items must be addressed to fully meet the learning outcome. The phrase *such as* indicates that the ensuing items are provided for illustrative purposes or clarification, and are not requirements that must be addressed to fully meet the learning outcome.

Achievement Indicators

Achievement indicators are samples of how students may demonstrate their achievement of the goals of a specific outcome. The range of samples provided is meant to reflect the scope of the specific outcome.

Specific curriculum outcomes represent the means by which students work toward accomplishing the general curriculum outcomes and ultimately, the essential graduation learnings.

Program Organization

Program Level	Course 1	Course 2	Course 3	Course 4
Advanced	Mathematics 1201	Mathematics 2200	Mathematics 3200	Mathematics 3208
Academic		Mathematics 2201	Mathematics 3201	
Applied	Mathematics 1202	Mathematics 2202	Mathematics 3202	

The applied program is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into the majority of trades and for direct entry into the workforce.

The academic and advanced programs are designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into post-secondary programs. Students who complete the advanced program will be better prepared for programs that require the study of calculus.

The programs aim to prepare students to make connections between mathematics and its applications and to become numerate adults, using mathematics to contribute to society.

Summary

The conceptual framework for Grades 10-12 Mathematics (p. 3) describes the nature of mathematics, mathematical processes and the mathematical concepts to be addressed. The components are not meant to stand alone. Activities that take place in the mathematics classroom should result from a problem-solving approach, be based on mathematical processes and lead students to an understanding of the nature of mathematics through specific knowledge, skills and attitudes among and between topics.

ASSESSMENT AND EVALUATION

Purposes of Assessment

What learning is assessed and evaluated, how it is assessed and evaluated, and how results are communicated send clear messages to students and others about what is really valued.

Assessment techniques are used to gather information for evaluation. Information gathered through assessment helps teachers determine students' strengths and needs in their achievement of mathematics and guides future instructional approaches.

Teachers are encouraged to be flexible in assessing the learning success of all students and to seek diverse ways in which students might demonstrate what they know and are able to do.

Evaluation involves the weighing of the assessment information against a standard in order to make an evaluation or judgment about student achievement.

Assessment has three interrelated purposes:

- assessment *for* learning to guide and inform instruction;
- assessment *as* learning to involve students in self-assessment and setting goals for their own learning; and
- assessment *of* learning to make judgements about student performance in relation to curriculum outcomes.

Assessment *for* Learning

Assessment *for* learning involves frequent, interactive assessments designed to make student understanding visible. This enables teachers to identify learning needs and adjust teaching accordingly. It is an ongoing process of teaching and learning.

Assessment *for* learning:

- requires the collection of data from a range of assessments as investigative tools to find out as mush as possible about what students know
- provides descriptive, specific and instructive feedback to students and parents regarding the next stage of learning
- actively engages students in their own learning as they assess themselves and understand how to improve performance.

Assessment *as* Learning

Assessment *as* learning actively involves students' reflection on their learning and monitoring of their own progress. It focuses on the role of the student as the critical connector between assessment and learning, thereby developing and supporting metacognition in students.

Assessment *as* learning:

- supports students in critically analysing their learning related to learning outcomes
- prompts students to consider how they can continue to improve their learning
- enables students to use information gathered to make adaptations to their learning processes and to develop new understandings.

Assessment *of* Learning

Assessment *of* learning involves strategies to confirm what students know, demonstrate whether or not they have met curriculum outcomes, or to certify proficiency and make decisions about students' future learning needs. Assessment *of* learning occurs at the end of a learning experience that contributes directly to reported results.

Traditionally, teachers relied on this type of assessment to make judgments about student performance by measuring learning after the fact and then reporting it to others. Used in conjunction with the other assessment processes previously outlined, however, assessment *of* learning is strengthened.

Assessment *of* learning:

- provides opportunities to report evidence to date of student achievement in relation to learning outcomes, to parents/guardians and other stakeholders
- confirms what students know and can do
- occurs at the end of a learning experience using a variety of tools.

Because the consequences of assessment *of* learning are often far-reaching, teachers have the responsibility of reporting student learning accurately and fairly, based on evidence obtained from a variety of contexts and applications.

Assessment Strategies

Assessment techniques should match the style of learning and instruction employed. Several options are suggested in this curriculum guide from which teachers may choose, depending on the curriculum outcomes, the class and school/district policies.

Observation (formal or informal)

This technique provides a way of gathering information fairly quickly while a lesson is in progress. When used formally, the student(s) would be aware of the observation and the criteria being assessed. Informally, it could be a frequent, but brief, check on a given criterion. Observation may offer information about the participation level of a student for a given task, use of a concrete model or application of a given process. The results may be recorded in the form of checklists, rating scales or brief written notes. It is important to plan in order that specific criteria are identified, suitable recording forms are ready, and all students are observed within a reasonable period of time.

Performance

This curriculum encourages learning through active participation. Many of the curriculum outcomes promote skills and their applications. In order for students to appreciate the importance of skill development, it is important that assessment provide feedback on the various skills. These may be the correct manner in which to use a manipulative, the ability to interpret and follow instructions, or to research, organize and present information. Assessing performance is most often achieved through observing the process.

Paper and Pencil

These techniques can be formative or summative. Whether as part of learning, or a final statement, students should know the expectations for the exercise and how it will be assessed. Written assignments and tests can be used to assess knowledge, understanding and application of concepts. They are less successful at assessing processes and attitudes. The purpose of the assessment should determine what form of paper and pencil exercise is used.

Journal

Journals provide an opportunity for students to express thoughts and ideas in a reflective way. By recording feelings, perceptions of success, and responses to new concepts, a student may be helped to identify his or her most effective learning style. Knowing how to learn in an effective way is powerful information. Journal entries also give indicators of developing attitudes to mathematical concepts, processes and skills, and how these may be applied in the context of society. Self-assessment, through a journal, permits a student to consider strengths and weaknesses, attitudes, interests and new ideas. Developing patterns may help in career decisions and choices of further study.

Interview

This curriculum promotes understanding and applying mathematics concepts. Interviewing a student allows the teacher to confirm that learning has taken place beyond simple factual recall. Discussion allows a student to display an ability to use information and clarify understanding. Interviews may be a brief discussion between teacher and student or they may be more extensive. Such conferences allow students to be proactive in displaying understanding. It is helpful for students to know which criteria will be used to assess formal interviews. This assessment technique provides an opportunity to students whose verbal presentation skills are stronger than their written skills.

Presentation

The curriculum includes outcomes that require students to analyze and interpret information, to be able to work in teams, and to communicate information. These activities are best displayed and assessed through presentations. These can be given orally, in written/pictorial form, by project summary, or by using electronic systems such as video or computer software. Whatever the level of complexity, or format used, it is important to consider the curriculum outcomes as a guide to assessing the presentation. The outcomes indicate the process, concepts and context for which a presentation is made.

Portfolio

Portfolios offer another option for assessing student progress in meeting curriculum outcomes over a more extended period of time. This form of assessment allows the student to be central to the process. There are decisions about the portfolio, and its contents, which can be made by the student. What is placed in the portfolio, the criteria for selection, how the portfolio is used, how and where it is stored, and how it is evaluated are some of the questions to consider when planning to collect and display student work in this way. The portfolio should provide a long-term record of growth in learning and skills. This record of growth is important for individual reflection and self-assessment, but it is also important to share with others. For all students, it is exciting to review a portfolio and see the record of development over time.

INSTRUCTIONAL FOCUS

Planning for Instruction

Consider the following when planning for instruction:

- Integration of the mathematical processes within each topic is expected.
- By decreasing emphasis on rote calculation, drill and practice, and the size of numbers used in paper and pencil calculations, more time is available for concept development.
- Problem solving, reasoning and connections are vital to increasing mathematical fluency and must be integrated throughout the program.
- There should be a balance among mental mathematics and estimation, paper and pencil exercises, and the use of technology, including calculators and computers. Concepts should be introduced using manipulatives and be developed concretely, pictorially and symbolically.
- Students bring a diversity of learning styles and cultural backgrounds to the classroom. They will be at varying developmental stages.

Teaching Sequence

The curriculum guide for Applied Mathematics 3202 is organized by units. This is only a suggested teaching order for the course. There are a number of combinations of sequences that would be appropriate.

Each two page spread lists the topic, general outcome, and specific outcome.

Instruction Time Per Unit

The suggested number of hours of instruction per unit is listed in the guide at the beginning of each unit. The number of suggested hours includes time for completing assessment activities, reviewing and evaluating. The timelines at the beginning of each unit are provided to assist in planning. The use of these timelines is not mandatory. However, it is mandatory that all outcomes are taught during the school year, so a long term plan is advised. Teaching of the outcomes is ongoing, and may be revisited as necessary.

Resources

The authorized resource for Newfoundland and Labrador students and teachers is *Math at Work 12* (McGraw-Hill Ryerson). Column four of the curriculum guide references *Math at Work 12* for this reason. Teachers may use any other resource, or combination of resources, to meet the required specific outcomes.

GENERAL AND SPECIFIC OUTCOMES

GENERAL AND SPECIFIC OUTCOMES WITH ACHIEVEMENT INDICATORS (pages 19-150)

This section presents general and specific outcomes with corresponding achievement indicators and is organized by unit. The list of indicators contained in this section is not intended to be exhaustive but rather to provide teachers with examples of evidence of understanding that may be used to determine whether or not students have achieved a given specific outcome. Teachers may use any number of these indicators or choose to use other indicators as evidence that the desired learning has been achieved. Achievement indicators should also help teachers form a clear picture of the intent and scope of each specific outcome.

Applied Mathematics 3202 is organized into seven units: *Measurement and Probability*, *Working With Data*, *Linear Relationships*, *Financial Decisions, Properties of Figures, Transformations*, and *Trigonometry*.

Measurement and Probability

Suggested Time: 16 Hours

Unit Overview

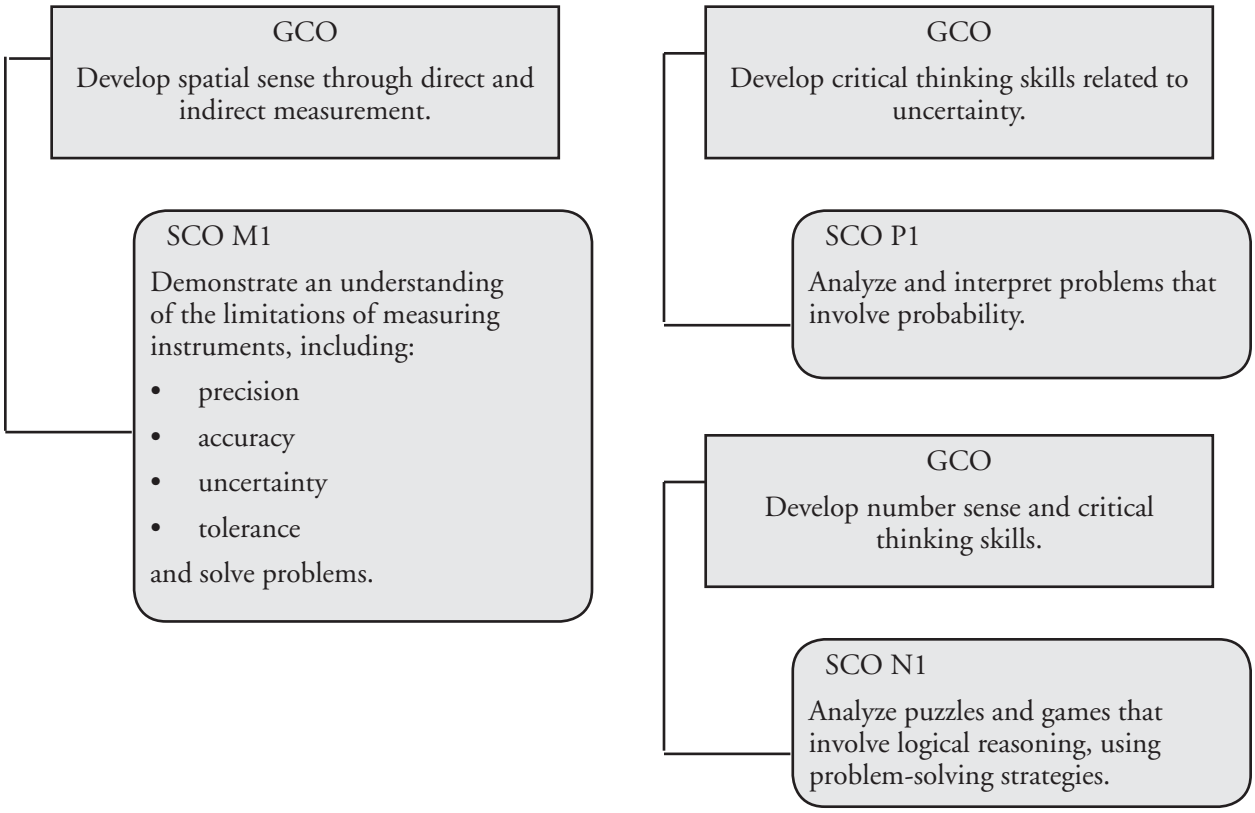
Focus and Context

The process of measurement is a link between mathematics and the physical world. In this unit, students are introduced to the concepts of accuracy, precision, uncertainty, and tolerance. Accuracy, precision, and uncertainty relate to the process of measuring. Tolerance acknowledges the variability that exists in the process of making products (e.g., fabricating, manufacturing, assembling, building).

Students will also analyze, interpret, and solve problems that involve probability. The focus is on calculations and uses of probability and odds.

In this unit, students investigate how calculations can help in making predictions. They express probability in words, as a fraction, as a decimal, and as a percent. They are introduced to odds and work with odds for and odds against. The probability of an event occurring is calculated based on a data set or based on the odds for or against.

Outcomes Framework



Mathematical Processes

[C] Communication
[CN] Connections
[ME] Mental Mathematics and Estimation
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

SCO Continuum

Mathematics 1202	Mathematics 2202	Mathematics 3202
Measurement		
M1. Demonstrate an understanding of the Système International (SI) by: • describing the relationships of the units for length, area, volume, capacity, mass and temperature • applying strategies to convert SI units to imperial units. [C, CN, ME, V] M2. Demonstrate an understanding of the imperial system by: • describing the relationships of the units for length, area, volume, capacity, mass and temperature • comparing the American and British imperial units for capacity • applying strategies to convert imperial units to SI units. [C, CN, ME, V]	M1. Solve problems that involve SI and imperial units in surface area measurements and verify the solution. [C, CN, ME, PS, V] M2. Solve problems that involve SI and imperial units in volume and capacity measurements. [C, CN, ME, PS, V]	M1. Demonstrate an understanding of the limitations of measuring instruments, including: • precision • accuracy • uncertainty • tolerance and solve problems. [C, PS, R, T, V]
Probability		
Not addressed	Not addressed	P1. Analyze and interpret problems that involve probability. [C, CN, PS, R]
Number		
G1. Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies. [C, CN, PS, R]	N1. Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies. [C, CN, PS, R]	N1. Analyze puzzles and games that involve logical reasoning, using problem-solving strategies. [C, CN, PS, R]

Measurement

Outcomes

Students will be expected to

M1 Demonstrate an understanding of the limitations of measuring instruments, including:

- **precision**
- **accuracy**
- **uncertainty**
- **tolerance**

and solve problems.

[C, PS, R, T, V]

Elaborations—Strategies for Learning and Teaching

In Mathematics 1202 and 2202, students explored the SI and imperial systems of measurement. They used various measuring devices to calculate measurements in SI and imperial units. Students worked with units for length, area, volume, capacity, mass and temperature. They also worked with unit conversions within and between both systems (M1, M2). In this unit, the focus is on the precision and accuracy of measurements.

Achievement Indicators:

M1.1 *Explain why, in a given context, a certain degree of precision is required.*

M1.2 *Explain why, in a given context, a certain degree of accuracy is required.*

Students should examine the definitions of precision and accuracy and compare their meanings. Accuracy is the degree of closeness of a measured value to the true value and depends on how carefully the measuring device is used. When using a measuring cup to measure a cup of sugar, certain assumptions about the accuracy of the measurement are being made:

- The manufacturer made sure that the measuring cup actually measures this amount.
- Other people can repeat the measurement with the same or a different measuring cup and get the same results.
- The accuracy depends on how carefully the device is used.

Precision is the degree of exactness to which a measurement is expressed and depends on the smallest scale division of the instrument used. The measuring instrument used will limit the degree of accuracy and precision. Some common measuring errors leading to inaccuracy include the use of incorrect units or reading the measuring device incorrectly.

General Outcome: Develop spatial sense through direct and indirect measurement.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to answer the following:

  If someone asked you your age, would you say "I'm 17" or "I'm 6500 days"? Which would be more precise? more accurate? Are there other answers you could give that are accurate and more precise?

  (M1.1, M1.2)

Resources/Notes

Authorized Resource

Math at Work 12

1.1 Accuracy and Precision

Student Book (SB): pp. 6-19

Teacher's Resource (TR): pp. 10-16

Measurement

Outcomes	Elaborations—Strategies for Learning and Teaching
Students will be expected to **M1 Continued ...** **Achievement Indicators:**	
M1.3 *Explain, using examples, the difference between precision and accuracy.*	Students should develop an understanding of the importance of accuracy and precision in daily life. If a contractor is buying baseboards for a room, for example, the measurements need to be accurate. However, when installing the baseboards, the measurements need to be precise. Present various situations and ask them to determine whether accuracy or precision is important. • A contractor develops material lists to provide reasonable estimates for jobs. • A seamstress hems a pair of pants. • A house painter determines how much paint is needed.
M1.4 *Analyze precision and accuracy in a contextual problem.*	They should analyze situations in which accuracy and precision are important. Cases such as the following could be discussed to illustrate why it is important to be accurate and/or precise in a variety of scenarios: • A person being treated for diabetes requires 2.5 units of insulin to account for the number of carbohydrates. He draws up 2.8 units of insulin to take. • A carpenter cuts a piece of crown moulding that is required to be 22.5°. Due to the tool being used, she can only cut to the nearest degree. When completed, the angle is noted to be 22°. Applying the concepts of precision and accuracy to real-world applications helps students to think critically about measurements in everyday life.
M1.5 *Compare the degree of accuracy of two given instruments used to measure the same attribute.*	Students should realize that different types of measuring devices may be more suitable for making certain measurements which may lead to varying degrees of accuracy. A ruler or tape measure is useful for accurately measuring most linear dimensions. However, a caliper may be more appropriate for identifying and measuring diameters.
M1.6 *Describe, using examples, the limitations of measuring instruments used in a specific trade or industry.*	The use of the measuring instrument depends on the situation. To distinguish between a standard 3-inch pipe and a standard 4-inch pipe, for example, a plumber may use a tape measure. A millwright constructing a non-standard size pipe may have to use a micrometer to measure its diameter. Understanding how the choice of measurement tools affects the accuracy and precision of a measurement will help students make informed choices when measuring.

General Outcome: Develop spatial sense through direct and indirect measurement.

Suggested Assessment Strategies

Journal

- Ask students to consider the statement *Measure twice; cut once*. They should discuss the implications for accuracy and precision and how it relates to different contexts (e.g., cooking, sewing, carpentry, electrical, plumbing).

(M1.1, M1.2, M1.3, M1.4)

Interview

- Students could interview a tradesperson, asking questions such as the following:

  (i) What tools do you use to measure?
  (ii) Why do you use different measuring tools? Is there one that you prefer?
  (iii) Which tools do you need to use to be accurate?
  (iv) Which tools do you need to use to be precise?
  (v) What factors do you consider when purchasing tools?

  Students should share their findings with the class.

(M1.4, M1.5, M1.6)

Performance

- Pose the following to students:

  A carpenter's tape typically shows $\frac{1}{32}$ divisions within the first foot. Beyond the first foot, the smallest division is $\frac{1}{16}$. Why is this the case?

  Using *Think-Pair-Share*, give individual students time to think about the question. Students then pair up with a partner to discuss their ideas. After pairs discuss, students share their ideas in a small-group or whole-class discussion.

  This could also be done as a *Think-Ink-Pair-Share* in which students are asked to write down their ideas before sharing with a partner.

(M1.7)

Resources/Notes

Authorized Resource

Math at Work 12

1.1 Accuracy and Precision

SB: pp. 6-19

TR: pp. 10-16

Measurement

Outcomes

Students will be expected to

M1 Continued ...

Achievement Indicators:

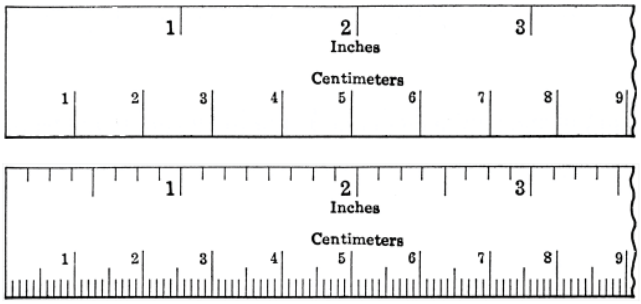
M1.7 *Relate the degree of precision to the uncertainty of a given measure.*

M1.8 *Calculate maximum and minimum values, using a given degree of tolerance in context.*

M1.9 *Solve a problem that involves precision, accuracy or tolerance.*

Elaborations—Strategies for Learning and Teaching

There is a degree of uncertainty when measuring due to the limitations of measuring devices. The margin of error of a measurement, if not stated, is half the precision of the measuring device. Students should consider which tool can provide a more precise measurement:



They should recognize that there is a greater degree of uncertainty when devices have less precision. The top ruler can measure to the nearest cm with certainty. The margin of error is 0.5 cm, so any measurement in the range of 0.5 cm below the actual measurement and 0.5 cm above the actual measurement would be acceptable. Using the bottom ruler, one-tenth of a cm can be measured with certainty, with a margin of error of one-twentieth of a cm.

Students should be aware that when making measurements, the degree of precision depends on the task. Sometimes a range of measurements, or tolerance, is acceptable. This tolerance acknowledges that measurements need to only be acceptable for the job they are intended to do. This allows for minimum and maximum allowable measurements. This is students' first exposure to the ± symbol. They are expected to calculate the maximum and minimum values given the acceptable tolerance.

Students should be provided with daily life examples of tolerance relative to measurement. Discuss with students the significance of the tolerance in situations such as the following:

- the dimensions of a door has a tolerance of 2 mm
- nail manufacturers produce 2-inch nails with a tolerance of $\frac{1}{8}$ inch
- a thermometer has a tolerance of 0.1°C

General Outcome: Develop spatial sense through direct and indirect measurement.

Suggested Assessment Strategies

Journal

- Ask students to respond to the following:

  When estimating materials for a job, 10-15% is often added to the estimate. Why is this a common practice?

  (M1.9)

Performance

- *Give Me Five* is an activity that provides students with an opportunity to individually and publicly reflect on their learning.

  Ask students: Why are manufacturing tolerances necessary? What can happen if they are not followed?

  Give time for students to quietly reflect before asking for five volunteers to share their reflections. A show of hands can also indicate how many students had a similar thought each time a student shares his or her reflection.

  (M1.8)

Resources/Notes

Authorized Resource

Math at Work 12

1.1 Accuracy and Precision

SB: pp. 6-19

TR: pp. 10-16

Measurement

Outcomes

Students will be expected to

M1 Continued ...

Achievement Indicators:

M1.8, M1.9 *Continued*

Elaborations—Strategies for Learning and Teaching

Students should realize that the tolerance will produce a range of values above and below the given measurement.

Most speedometers, for example, have tolerances of ± 10%, mainly due to variations in tire diameter. Sources of error due to tire diameter variations are wear, temperature, pressure, vehicle load, and minimal tire size. Students may be interested to know that vehicle manufacturers usually calibrate speedometers to read high by an amount equal to the average error, to ensure that their speedometers never indicate a lower speed than the actual speed of the vehicle.

Students are expected to apply the concepts of accuracy, precision and tolerance in order to solve contextual problems. They may be presented with a tolerance, required to calculate an acceptable range of values, and then asked to comment on the accuracy or precision of a given measurement. They should solve problems such as the following:

A dairy farmer prepares 15 mL of antibiotic in a syringe marked in 1-mL increments. The tolerance is ± 0.5 mL. What are the maximum and minimum allowable measurements?

Discuss why it is important for tradespeople to be familiar with the idea of tolerances. Time and materials will not be wasted in trying to make items that are dimensionally exact, but that function according to the requirements of their use.

General Outcome: Develop spatial sense through direct and indirect measurement.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to determine the maximum and minimum measures allowable, based on the given tolerance.

  (i) $1.2699'' \pm 0.0009''$
  (ii) $2.000'' \pm 0.002''$
  (iii) $10.203 \text{ mm} \pm 0.024 \text{ mm}$
  (iv) $64.86 \text{ mm} \pm 0.03 \text{ mm}$

  (M1.8)

- A machine part can measure 44.31 ± 0.01 mm. Ask students to determine the acceptable maximum and minimum dimensions.

  (M1.8)

- Greg competed in a snowboard race and finished with a time of 1:16.36.

  Ask students to answer the following:

  (i) What is the level of precision of the time measurement?
  (ii) What is the uncertainty of the measurement?

  (M1.9)

Journal

- Ask students to reflect on what they have learned about measurement. Prompting questions might include:
  - What is the difference between an accurate measurement and a precise measurement?
  - How do you know how precise a measurement is?
  - What is a measurement tolerance?

  (M1.3, M1.7, M1.8)

- Ask students to respond to the following:

  (i) Discuss why vehicle manufacturers ensure that the speedometer never indicates a lower speed than the actual speed of the vehicle.

  (M1.8)

Resources/Notes

Authorized Resource

Math at Work 12

1.1 Accuracy and Precision

SB: pp. 6-19

TR: pp. 10-16

Probability

Outcomes

Students will be expected to

P1 Analyze and interpret problems that involve probability.

[C, CN, PS, R]

Achievement Indicators:

P1.1 *Calculate the probability of an event based on a data set.*

P1.2 *Express a given probability as a fraction, decimal and percent and in a statement.*

Elaborations—Strategies for Learning and Teaching

Students have previously expressed probabilities as ratios, fractions and percents (7SP4) and compared theoretical and experimental probabilities of independent events (7SP6). In Grade 8, they created and solved problems using probabilities (8SP2), including the use of tree diagrams and simulations. In this unit, they will analyze problems that involve probability and distinguish between probability and odds. The focus of student work should be on independent events. They will calculate probability and odds, and use this information to make decisions. Students should recognize that many day to day decisions are based on the combination of probability and subjective judgments.

When there is uncertainty about the occurrence of an event, students can attempt to measure the chances of it happening with probability. As an introduction, ask students to consider the following:

- Will the Ice Caps or the Pirates be more likely to win the game?
- What is the chance of drawing a king from a deck of cards?
- What is the possibility of rain today?
- What are the chances of getting tails in one toss of a coin?

Students need to be reminded that probability is determined by dividing the number of favorable outcomes by the total number of possible outcomes. This results in a value ranging from 0 to 1, where 0 refers to that event never happening and 1 refers to the event always happening. Students should distinguish between theoretical and experimental probability. This could be demonstrated through a coin flip activity, where they compare their actual results with the theoretical probability of obtaining heads or tails. Ask students to consider the following:

Every time a coin is flipped, there is a 50/50 chance of landing on heads or tails. So, if a coin is flipped 50 times, the result should be 25 heads and 25 tails.

Ask if they agree with this. This can begin a discussion about theoretical probability, which can then be tested. Have small groups flip a coin 50 times and tally heads and tails. Then compare the experimental probability calculated from their data with the theoretical probability.

Online or graphing calculator probability simulators could be used to generate data for large numbers of trials.

Probability should be expressed as a fraction, decimal, and percent. Converting between these three representations should be reviewed.

General Outcome: Develop critical thinking skills related to uncertainty.

Suggested Assessment Strategies

Performance

- Students could work with a partner to create an avatar for a video game. Their avatar will have three characteristics: gender (male or female), hair color (black, brown, red, blonde) and eye color (blue, green, brown). They will use a die to determine the characteristics based on the following:
  - The first roll identifies the gender. Roll a 1, 2, or 3 and the avatar will be male, otherwise it is female.
  - The second roll identifies the hair color. A 1 or 2 represents black hair, a 3 or 4 represents brown hair, a 5 represents red hair and a 6 represents blonde hair.
  - The third roll identifies eye color. A 1 or 2 is blue, a 3 or 4 is green and 5 or 6 is brown.

  Students can draw a picture of their avatar and determine the:

  (i) probability of being male
  (ii) probability of having brown hair
  (iii) probability of having blue eyes

  (P1.1)

Paper and Pencil

- Ask students to copy and complete the following table for each of the given probabilities.

Ways of Expressing Probability			
Fraction	Decimal	Percent	Words
$\frac{1}{2}$			
	0.60		
			2 in 3
		75%	

(P1.2)

- Ask students to consider the following statements and respond to the questions that follow:

  There is a 10% chance of snow.

  Four out of five dentists recommend this tooth brush.

  Goalie Roberto Loungo's saves statistic was 0.927 at the 2010 Olympic Winter Games.

  What is the probability expressed in each of the other forms?

  (P1.2)

Resources/Notes

Authorized Resource

Math at Work 12

1.2 Probability and Odds

SB: pp. 20-29

TR: pp. 17-24

1.3 Theoretical and Experimental Probability

SB: pp. 30-41

TR: pp. 25-33

Probability

Outcomes

Students will be expected to

P1 Continued ...

Achievement Indicators:

P1.1, P1.2 *Continued*

P1.3 *Explain the difference between odds and probability.*

P1.4 *Determine the probability of an event, given the odds for or against.*

Elaborations—Strategies for Learning and Teaching

Probability questions involving playing cards are common. It might help to post a reminder such as the following in the classroom and have decks of cards available for student use:

A regular deck of cards has:

52 cards total

26 red (13 diamonds, 13 hearts) and 26 black (13 spades, 13 clubs)

Each of the 4 suits has the cards 2-10, J, Q, K, A.

Although students have been introduced to probability in previous grades, this will be their first formal introduction to odds. It is important to highlight the difference between probability and odds. While probability compares the number of favorable outcomes to the total number of outcomes, odds compares the number of favorable outcomes to the number of unfavorable outcomes. The probability of rolling a 3 on a six-sided die is $\frac{1}{6}$. The odds for rolling a 3 on a six-sided die are 1:5. Odds is a part-to-part comparison. Since a fraction compares a part to a whole, odds can only be expressed as a ratio.

Students should be exposed to situations involving odds for and odds against. Odds for equals the ratio of favourable outcomes to unfavourable outcomes. Odds against equals the ratio of unfavourable outcomes to favourable outcomes. Students should realize that the ratio of the odds against an event is the reverse of the ratio for an event. If the odds for selecting a red marble from a jar is 5:3, for example, then the odds against selecting a red marble is 3:5.

Given the odds for or against, students should calculate the probability of an event. To determine the probability of an event occurring if the odds against the event are 4:1, students must realize that there are 4 unfavourable outcomes and 1 favourable outcome. The probability of the event occurring, therefore, is 1 in 5.

General Outcome: Develop critical thinking skills related to uncertainty.

Suggested Assessment Strategies

Paper and Pencil

- In groups of two, ask students to consider the following scenarios and decide whether probability or odds would be a more useful way of describing the data.
  (i) The likelihood of a defective product made on an assembly line: in past tests 1450 were correctly made and 25 were defective
  (ii) The likelihood of the school soccer team winning the next game: the school team won 4 out of their last 6 games

  (P1.3)

- Ask students to determine the probability of each event:
  (i) Odds in favour of the event are 1:3.
  (ii) Odds against the event are 5:1.
  (iii) Odds in favour of the event are 50:1.
  (iv) Odds against the event are 1:1.

  (P1.4)

Resources/Notes

Authorized Resource

Math at Work 12

1.2 Probability and Odds

SB: pp. 20-29

TR: pp. 17-24

1.3 Theoretical and Experimental Probability

SB: pp. 30-41

TR: pp. 25-33

Probability

Outcomes

Students will be expected to

P1 Continued ...

Achievement Indicators:

P1.5 *Explain, using examples, how decisions may be based on a combination of theoretical probability calculations, experimental results and subjective judgements.*

P1.6 *Describe and explain the applications of probability.*

P1.7 *Solve a contextual problem that involves odds or probability.*

Elaborations—Strategies for Learning and Teaching

Theoretical probabilities are those that result from theory (what should happen mathematically), while experimental probabilities are those that result from experiments or repeated trials of performing the event. Students should think about how decision making is affected by a combination of probability and subjective judgements. Ask them to consider the variety of strategies people use when choosing their lottery numbers. Some use the same numbers for repeated lotteries, others use past frequencies to select their numbers, while others allow their numbers to be randomly selected.

A discussion of games of chance could lead to an explanation of how theoretical probability, experimental probability and subjective judgement would play a role in decisions made when playing such games.

Calculations of probability are always based on assumptions. Students should be encouraged to identify and examine the assumptions to help them determine whether the calculated probability is meaningful when making a decision. Students should engage in evaluating situations that lend themselves to reasonably accurate predictions, those that are questionable, and those for which the unknowns are not quantifiable. Road accidents with/without seatbelts are good examples for safe prediction. Health professionals predicting that people of lower socioeconomic status will have more health problems is a more questionable situation.

Expose students to a variety of situations involving odds and probability. These can include, but are not limited to, selecting cards/marbles, rolling die and flipping coins. Many games of chance use odds and probability. Examples can also be found or created based on school statistics (e.g., the number of biology students expected in Grade 12) or sports (e.g., the chance of a team winning a championship). This would also be a good opportunity for students to conduct in-class surveys and calculate odds or probability based on results.

General Outcome: Develop critical thinking skills related to uncertainty.

Suggested Assessment Strategies

Journal

- Ask students to respond to the following:

  The Rogers Centre, which is home to the Toronto Blue Jays, has the world's first fully retractable roof. The decision to host an event with an open or closed roof is based upon detailed weather information gathered by the roof technicians. If the technicians were given the following forecast, do you think they would leave the roof open? Use probability to justify your choice.

Short Term Forecast Update

	Tuesday Afternoon
	Risk of thunderstorms
Temp.	36°C
Feels Like	43
Wind	W 25km/h
Relative Humidity	44 %
P.O.P.	40 %
Rain	close to 1 mm

(P1.5, P1.6)

Paper and Pencil

- Ask students to answer the questions that follow, based on the table.

	Tuesday Afternoon	Tuesday Evening	Tuesday Overnight	Wednesday Morning	Wednesday Afternoon
	Cloudy periods	Variable cloudiness	Variable cloudiness	Cloudy with showers	Cloudy with showers
Temp.	23°C	21°C	14°C	16°C	19°C
Wind	S 15km/h	S 10km/h	SE 5km/h	SE 10km/h	NE 10km/h
Relative Humidity	46 %	53 %	72 %	77 %	77 %
P.O.P.	20 %	20 %	30 %	40 %	60 %
Rain	-	-	-	less than 1 mm	close to 1 mm

(i) What is the probability of precipitation on Wednesday morning?
(ii) What are the odds of precipitation on Tuesday evening?
(iii) What is the probability against precipitation on Tuesday afternoon? (P1.7)

Resources/Notes

Authorized Resource

Math at Work 12

1.3 Theoretical and Experimental Probability

SB: pp. 30-41

TR: pp. 25-33

Web Link

http://illuminations.nctm.org/LessonDetail.aspx?ID=L248

Instructions for a version of the game SKUNK, along with follow-up activities, can be found on the NCTM website.

1.4 Working With Probability

SB: pp. 42-53

TR: pp. 34-40

Number

Outcomes

Students will be expected to

N1 Analyze puzzles and games that involve logical reasoning, using problem-solving strategies.

[C, CN, PS, R]

Elaborations—Strategies for Learning and Teaching

This outcome is intended to be integrated throughout the course by using puzzles and games with a focus on logical reasoning. In Mathematics 1202 (G1), students applied problem-solving strategies to analyze puzzles and games that involved spatial reasoning. In Mathematics 2202 (N1), the focus was on puzzles and games that involve numerical reasoning.

A variety of puzzles and games, such as board games, online puzzles and games, appropriate selections for gaming systems, and paper and pencil puzzles should be used. It is not intended that the activities be taught in a block of time, but rather explored periodically during the year. Timing and integration of this outcome should be included in teacher planning throughout the course. Students could be exposed to three or four games at different times, whether it be at the beginning or end of each unit, or a set "game day". They could also engage in a game when they are finished other work. As students work through the different games and puzzles, they will begin to develop effective strategies for solving the puzzle or winning the game.

Games provide opportunities for building self-concept, enhancing reasoning and decision making, and devloping positive attitudes towards mathematics through reducing the fear of failure and error. In comparison to more formal activities, greater learning can occur through games due to increased interaction between students and opportunities to explore intuitive ideas and problem-solving strategies. Students' thinking often becomes apparent through the actions and decisions they make during a game, so teachers have the opportunity to formatively assess learning in a non-threatening situation.

It may be beneficial to discuss with students some of the benefits of engaging in these activities:

- improved critical thinking skills
- increased attention span
- improved concentration
- slower rate of memory decline
- improved ability to relax the mind, which can improve productivity.

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies

Observation

- Playing games creates dialogue and provides a tool for informal assessment. Stations could be set up with one or two games at each centre. Teachers should circulate among the groups and assess students' understanding. Puzzles and games involving logical reasoning could include:

  (i) chess
  (ii) logic puzzles
  (iii) Minecraft
  (iv) Sudoku™
  (v) Clue™
  (vi) Monopoly™
  (vii) Game of Life™
  (viii) Mancala

  Students can work through the puzzles or games individually or with a partner. They could record their progress in a table such as the one shown below.

Puzzle	Solved?	Strategy	Comments/Hints

Journal

- Ask students to write about the puzzles and games they found interesting, and why.

Performance

- Using a game or puzzle of their choice, ask students to write their own description of the game/puzzle, the rules of play, and helpful hints. They should give the information to another classmate as they play the game.

Resources/Notes

Authorized Resource

Math at Work 12

Each chapter has a section called *Games and Puzzles* that can be used to support this outcome. The reference appears in Resources/Notes where appropriate.

Number

Outcomes

Students will be expected to

N1 Continued ...

Achievement Indicators:

N1.1 *Deterimne, explain and verify a strategy to solve a puzzle or to win a game; e.g.,*

- *guess and check*
- *look for a pattern*
- *make a systematic list*
- *draw or model*
- *eliminate possibilities*
- *simplify the original problem*
- *work backward*
- *develop alternative approaches.*

N1.2 *Identify and correct errors in a solution to a puzzle or in a strategy for winning a game.*

Elaborations—Strategies for Learning and Teaching

Students need time to play and enjoy the game before analysis begins. They can discuss the game, determine the winning strategies and explain these strategies through demonstration, oral explanation or in writing.

Problem-solving strategies will vary depending on the puzzle or game. Some students will explain their stategy by working backward. looking for a pattern, using guess and check, or by eliminating possibilities, while others will plot their moves by trying to anticipate their opponents' moves. As students play games and analyze strategies, they explore mathematical ideas and compare different strategies for efficiency.

There may be situations where students are able to play the game and solve problems but are unable to determine a winning strategy. Teachers could participate with the group and think through the strategies out loud so the group can hear the reasoning for selected moves. Ask the group's opinions about the moves in the game and facilitate discussions around each of the players' moves and strategies.

When introducing games, students will need to understand the rules and procedures of the game. Consider the following:

- Introduce the game to one group of students while others are completing individual work. Then divide the whole class into groups, putting a student from the first group into each of the other groups to teach them the game.
- Choose students to play the game as a demonstration, possibly with assistance in decision making from the whole class.
- Divide the class into groups. Play the game with the whole class, with each group acting as a single player.

Puzzles and games stations are a good way to organize these activities. Consider the following tips when creating the stations:

- Some stations may require multiple games, while other stations may involve one game that requires more time to play.
- Divide students into small groups. At regular intervals, have students rotate to the next station.
- As students play a game, pose questions about the strategies they are using.
- Engage students in post-game discussion.

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies

Performance

- In the game *High or Low*?, students guess whether the next card is higher or lower than the last one dealt. They could also be asked to determine the exact probability that the next card will be higher or lower or the same suit.

  Divide the class into small groups, giving each one no more than 10 regular playing cards. Each group should have a different set of cards and probabilities to figure out. Students can use a graphic organizer such as the one shown here to keep track of the play.

Card	Probability HIGH?	Probability LOW?	Probability EQUAL?	Your Guess?	Correct Guess?

  Once the activity is complete, use reflection questions such as the following:

  (i) How many of your group's guesses were right? How do you think your group did overall?
  (ii) Was this game easier or more difficult than you thought before you started? Explain.
  (iii) How would the game change if you put each card back into the deck and shuffled every time before dealing a new card?

  Students could complete an exit card for this activity.

  How could we change this game to make it easier? more challenging?

  Students could also be asked to write probabilities as a percent. In this case, give less cards to each group, as it will take more time.

Resources/Notes

Authorized Resource

Math at Work 12

Games and Puzzles: Nim

SB: p. 59

TR: p. 45

Number

Outcomes

Students will be expected to

N1 Continued ...

Achievement Indicators:

N1.1, N1.2 *Continued*

Elaborations—Strategies for Learning and Teaching

As students work through the games and puzzles, it is a good idea to keep a checklist of the games and puzzles each of them is working on. Students could work in groups where each member has been exposed to a different game or puzzle. Ask the group to do the following:

- Explain the rules of the game in your own words to the other group members.
- Give a brief demonstration of how the game is played.
- Give advice to other students trying to solve the puzzle or play the game.

Invite students to bring in their own games and puzzles involving logical reasoning. This may involve bringing in board games from home or searching the Internet to find a game or puzzle that interests them. Students can introduce this game/puzzle by providing information such as the following:

- What is the puzzle or game, and where did you find it?
- Describe the puzzle or game. Why did you select it?
- Describe the objective of the game and the rules of play.
- What strategy would you use to solve the puzzle or play the game?

Logic mazes have different rules that determine how you can navigate through them. In some types of logic mazes, it is possible for the rules to change while you are in the maze. Students could be exposed to some of the following types of logic mazes:

- Number mazes, in which a grid of numbers is navigated by travelling the number shown on the current square
- Eyeball mazes, where you can jump from one square to another based on the colour or shape of the symbol on that square
- Multi-state mazes, in which the rules for navigation change depending on how the maze has been navigated.

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies

Performance

- Students could play the Native American *Stick Game*.

  Materials:

  - 4 popsicle sticks
  - markers
  - 50 toothpicks or beans to keep score

  Students decorate each popsicle stick on one side with markers. They should use a different pattern or color for each one. The decorated side is the "face" of the piece.

  Players take turns. Each player holds the sticks in one hand and lets them fall to the ground or the table.

  Scoring:

  - All four up = 5 points
  - Three up and one down = 2 points
  - Two up and two down = 1 point
  - One up and three down = 2 points
  - All four down = 5 points

  They count the number of points and take that many toothpicks or beans from the pile. The player with the most toothpicks or beans at the end of the agreed-upon number of rounds is the winner.

  Once the game is over, ask students to consider:

  (i) Is this a fair way to score the game?
  (ii) Describe a better way to score.

  There are sixteen different ways that four sticks can fall. Three ways are shown in the table.

#1	#2	#3	#4
up	up	up	up
down	up	up	up
up	down	up	up

  Ask students to finish this table, and determine the theoretical probability of two sticks landing up and two sticks landing down.

Resources/Notes

Authorized Resource

Math at Work 12

Games and Puzzles: Nim

SB: p. 59

TR: p. 45

Web Link

www.logicmazes.com

This site contains various types of logic mazes.

Number

Outcomes

Students will be expected to

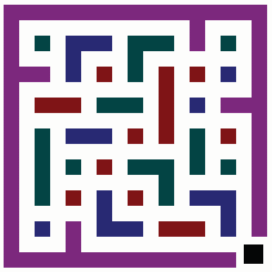
N1 Continued ...

Achievement Indicators:

N1.1, N1.2 *Continued*

Elaborations—Strategies for Learning and Teaching

A No-Left-Turn maze would be a good place for students to start. This one has a certain intuitive quality, perhaps because when we drive, we often plan a route that avoids left turns.



Logic puzzles are often based on statements which contain clues to the solution of the problem. These clues may be positive or negative (i.e., they may tell you part of the answer or tell you what the answer is not). Discuss with students that it is just as important to know what can be eliminated as it is to find the actual correct answer. To keep track of these possibilities (and non-possibilities), students could use organizers such as tables, lists or Venn diagrams.

To introduce logic puzzles, begin with one that has only a small number of variables:

Amanda, Kim, Alex and Sarah each have different coloured cars. One car is red, one is blue, one is white and the other is black. Who owns which car?

Clue 1: Amanda's car is not red or white.

Clue 2: Kim's car is not blue or white.

Clue 3: Alex's car is not black or blue.

Clue 4: Sarah's car is red.

Provide students with an organized list similar to the following:

	Amanda	Kim	Alex	Sarah
red				
blue				
white				
black				

Students can place an X in a box to eliminate a possibility and a √ in a box they know is correct.

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies

Paper and Pencil

- Three containers can hold 19 L, 13 L and 7 L of water, respectively. The 19-L container is empty. The other two are full. Ask students how they can measure 8 L of water using no other container and no other water supply.

- Ask students to answer the following:

  You have 3 boxes of fruit.

  One contains just apples, one contains just oranges, and one contains both apples and oranges.

  Each box is labelled as shown:

  However, it is known that none of the boxes are labelled correctly. How can you label the boxes correctly if you are only allowed to take and look at just one piece of fruit from one of the boxes?

- Students could answer a Knights and Knaves puzzle:

  A very special island is inhabited only by knights and knaves. Knights always tell the truth, and knaves always lie.

  You meet two inhabitants: Zoey and Mel. Zoey tells you that Mel is a knave. Mel says that neither she nor Zoey are knaves.

  Can you determine who is a knight and who is a knave?

Resources/Notes

Authorized Resource

Math at Work 12

Games and Puzzles: Nim

SB: p. 59

TR: p. 45

Number

Outcomes

Students will be expected to

N1 Continued ...

Achievement Indicators:

N1.1, N1.2 *Continued*

Elaborations—Strategies for Learning and Teaching

Work through problems with a greater number of variables as a class first. For the example below, students could work in 6 small groups with each group getting a clue.

Four friends met on Saturday morning for breakfast. Each friend ordered a different drink and breakfast meal during their visit and, when it was time to leave, each got a different drink to go. Determine the first name of the friend, the drink and meal each ordered for breakfast, and the drink each ordered to go.

Clues:

- Brenda had waffles but not an espresso.
- The friend who ordered the pancakes also ordered decaf coffee to go but didn't have cranberry juice.
- The woman who ordered the omelet had water to drink but she wasn't Amy.
- The two friends who ordered juice were Emily and the friend who ordered an egg sandwich.
- The friend who had a cappuccino didn't order orange juice.
- Melony ordered a hot tea to go.

Display an organzied list and ask each group to contribute to completing the table by sharing their clues.

	cranberry juice	milk	orange juice	water	egg sandwich	omelet	pancakes	waffles	cappuccino	decaf coffee	espresso	hot tea
Amy												
Brenda												
Emily												
Melony												
cappuccino												
decaf coffee												
espresso												
hot tea												
egg sandwich												
omelet												
pancakes												
waffles												

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies

Paper and Pencil

- Students work in groups of six. The number of group members should match the number of clues. Each group has an envelope containing the clues for a logic puzzle. Students are only allowed to look at their own clue. They may share their clue with the other group members by telling them what is on it, but they can't show it to anyone else.

  Sample Puzzle:

  Rachel, Linda and Eve were friends sitting in a circle on the grass.

  - Rachel passed 3 chocolate chip cookies to the person in blue.
  - Eve passed 3 macaroons to the person who passed her cookies to the person wearing green.
  - Each person passed 3 cookies to the friend on her left.
  - Rachel, Linda and Eve were dressed in red, blue and green, but not necessarily in that order.
  - The person who was wearing green did not get a macaroon.
  - The person wearing red passed along 3 oatmeal cookies.

  Who wore which colour?

  Students work together to solve the puzzle.

  While observing the activity, focus on questions such as:

  (i) How did your group get started?
  (ii) Were there any challenges?
  (iii) Which clue did you find most helpful?
  (iv) Were there any clues that didn't help you solve the puzzle?
  (v) What might you do differently next time?

- Students could work together to solve the following puzzle:

  A man wishes to cross the river with a wolf, a goat and some hay. He has a small boat, but unfortunately, he can only take one thing across at a time. What is worse, if he leaves the wolf and the goat alone together, the wolf will eat the goat, and if he leaves the goat with the hay, the goat will eat the hay. How does he do it?

Resources/Notes

Authorized Resource

Math at Work 12

Games and Puzzles: Nim

SB: p. 59

TR: p. 45

Number

Outcomes

Students will be expected to

N1 Continued ...

Achievement Indicator:

N1.3 *Create a variation on a puzzle or a game, and describe a strategy for solving a puzzle or winning the game.*

Elaborations—Strategies for Learning and Teaching

As an alternative, students may have an idea for a game or puzzle that would challenge their classmates. To create a game, they could use the rules of an existing game, but use different materials or add extra materials. They could also use the idea for a game and change the rules. Another option is to use a board game and add math tasks to it. Rather than writing tasks directly onto the boards, they can place coloured stickers on certain spaces and make up colour-coded cards with questions. A game such as Snakes and Ladders™, for example, can be modified to Operation Snakes and Ladders. The board can be used with two dice. On each turn, to determine the number of spaces to move, the player has the option of multiplying, dividing, adding or subtracting the two numbers, with a maximum answer of twenty.

The following guiding questions could be used to help students evaluate their games.

- Can the game be completed in a short time?
- Is there an element of chance built in?
- Are there strategies which can be developed to improve the likelihood of winning?

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies

Observation

- As students play games or puzzles, ask probing questions and listen to students' responses. Record the different strategies and use these strategies for class discussion. Possible discussion starters include:

  (i) Thumbs up if you liked the game, thumbs sideways if it was okay, and thumbs down if you didn't like it. What did you like or dislike about it? Why?
  (ii) What did you notice while playing the game?
  (iii) Did you make any choices while playing?
  (iv) Did anyone figure out a way to quickly find a solution?

Journal

- Ask students to respond to the following:

  Your friend is having trouble starting the Sudoku puzzle below. Find three of the missing numbers and explain the strategy you used to help him get started.

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

Resources/Notes

Authorized Resource

Math at Work 12

Chapter Project: Design a Game

SB: p. 58

TR: pp. 43-44

Working With Data

Suggested Time: 13 Hours

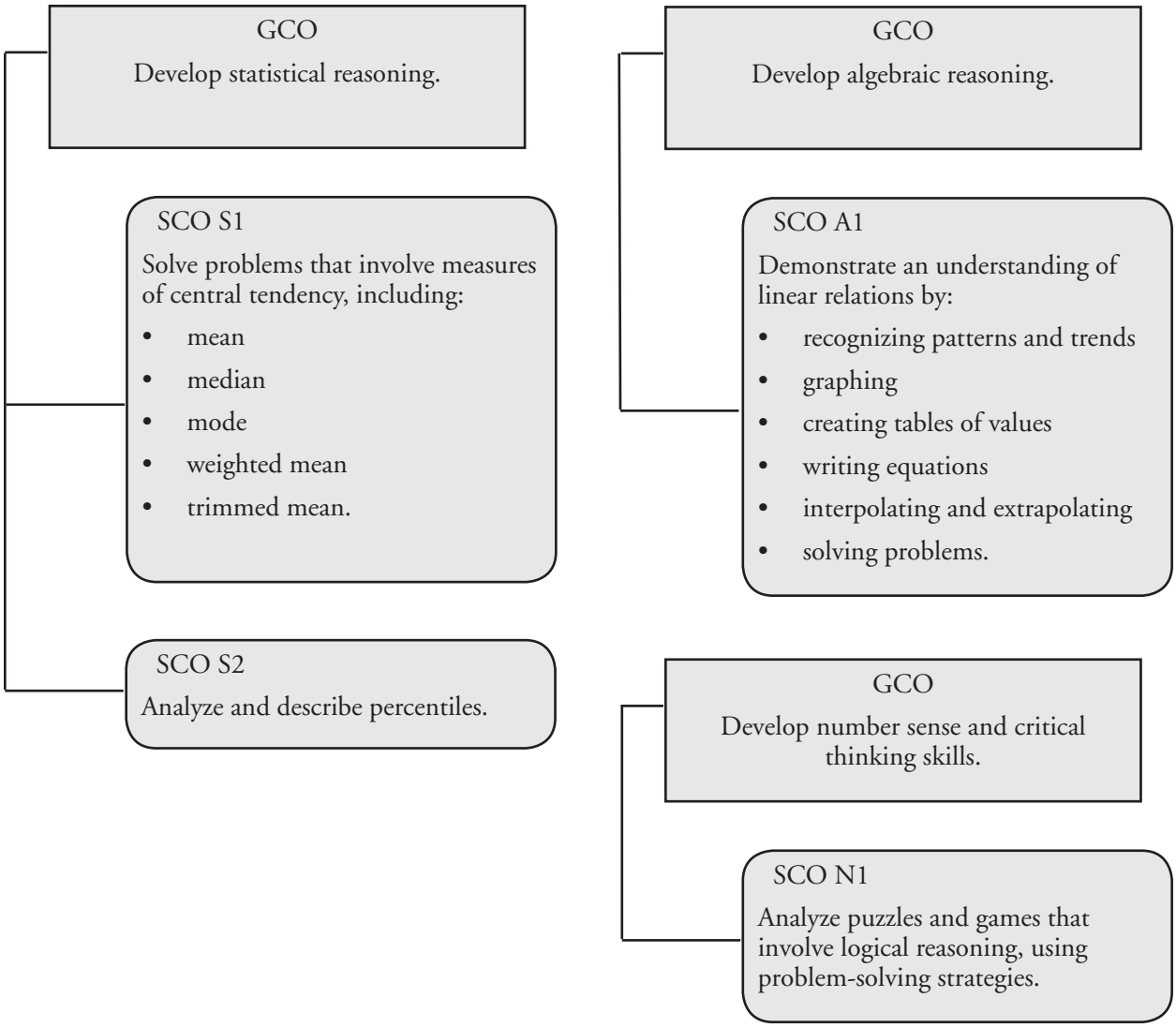
Unit Overview

Focus and Context

This unit provides a review of the concepts of mean, median and mode, which were introduced in Grade 7. It extends prior student learning to discussion of trimmed and weighted means and the effects of outliers. Percentile rankings are also introduced.

The mean, the median, and the mode are the most basic concepts in statistical thinking. It is important that students have an understanding of these concepts so that they can become critical observers of data they encounter. It is also important that students are aware of the different representations that can be used to present data and realize when one measure gives a better representation of the data.

Outcomes Framework



Mathematical Processes

[C] Communication
[CN] Connections
[ME] Mathematical Modeling and Estimation
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

SCO Continuum

Mathematics 1202	Mathematics 2202	Mathematics 3202
Statistics		
not addressed	S1. Solve problems that involve creating and interpreting graphs, including: • bar graphs • histograms • line graphs • circle graphs. [C, CN, PS, R, T, V]	S1. Solve problems that involve measures of central tendency, including: • mean • median • mode • weighted mean • trimmed mean. [C, CN, PS, R, T, V] S2. Analyze and describe percentiles. [C, CN, PS, R]
Algebra		
not addressed	not addressed	A1. Demonstrate an understanding of linear relations by: • recognizing patterns and trends • graphing • creating tables of values • writing equations • interpolating and extrapolating • solving problems. [CN, PS, R, T, V]
Number		
G1. Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies. [C, CN, PS, R]	N1. Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies. [C, CN, PS, R]	N1. Analyze puzzles and games that involve logical reasoning, using problem-solving strategies. [C, CN, PS, R]

Statistics

Outcomes

Students will be expected to

S1 Solve problems that involve measures of central tendency, including:

- **mean**
- **median**
- **mode**
- **weighted mean**
- **trimmed mean.**

[C, CN, PS, R]

Achievement Indicators:

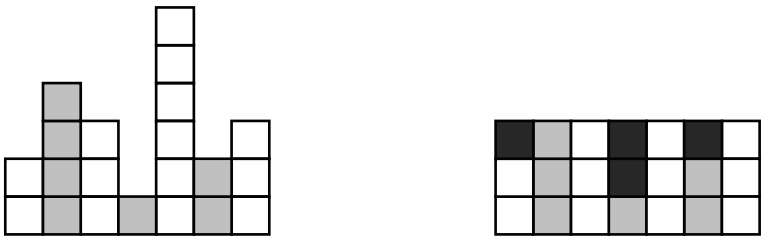
S1.1 *Determine the mean, median and mode for a set of data.*

S1.2 *Solve a contextual problem that involves measures of central tendency.*

Elaborations—Strategies for Learning and Teaching

Measures of central tendency allow a set of data to be described with a single meaningful number. In Grade 7, students calculated measures of central tendency and used them to solve contextual problems (7SP1, 7SP2). These topics will be further explored in this unit. The concepts of weighted mean and trimmed mean will be introduced, calculated and analyzed in relevant situations. Students will evaluate when it is more appropriate to use a trimmed or weighted mean rather than just the mean.

A review of how to calculate each measure of central tendency will be needed. One activity that could be explored to determine the mean of a data set involves using blocks. The blocks can be rearranged so each column has equal height. When using this strategy, ensure the data sets used have a small number of elements with low numbers, and that they balance to a whole number. The data set 2, 4, 3, 1, 6, 2, 3, for example, has a mean of 3, as seen in the diagram below:



Using an algorithm, students can compute the mean by adding all the numbers in a data set and dividing the sum by the number of elements added.

Rolls of tickets can be used to review the concept of finding the median of a data set. On a strip of tickets, students write a data value on each ticket in ascending order. If there are an odd number of tickets the strip is folded at the median.

The following set with an odd number of values has a median of 56.

12	16	31	42	48	56	63	64	78	83	91

The data set, containing an even number of values, has a median of 50.

12	16	27	31	42	46	54	56	63	64	78	82

General Outcome: Develop statistical reasoning.

Suggested Assessment Strategies

Performance

- Students can create a foldable to define and give examples of each of the measures of central tendency. On each of the outside panels, have students define mean, median, or mode. On the corresponding inside panel, they can create an example and solve a problem using the corresponding measure of central tendency on the front.

  (S1.1)

- Ask students to collect the heights of the players of the school basketball team. They should write each height on an index card and order the cards to determine the median and mode heights. They could also do this with a favourite professional team or an all star team.

  (S1.1)

Paper and Pencil

- A class set of data could be generated by asking each student in the class to record the approximate time it takes him or her to get to school. This can be presented as a grafitti wall with sticky notes. The sticky notes can then be rearranged to help students visualize mode and median. Students can then use the data to answer the following questions:

  (i) Calculate the mean time students in the class spend travelling to school each day.

  (ii) Calculate the median and mode for time spent travelling to school. How do they differ? Which is more useful? Explain your reasoning.

  (S1.1, S1.2)

Resources/Notes

Authorized Resource

Math at Work 12

2.1 Measures of Central Tendency

Student Book (SB): pp. 64-79

Teacher's Resource (TR): pp. 54-63

Statistics

Outcomes

Students will be expected to

S1 Continued ...

Achievement Indicators:

S1.1, S1.2 *Continued*

Elaborations—Strategies for Learning and Teaching

Data could also be displayed in a stem and leaf plot. The previous odd-numbered set is shown below:

Stem	Leaf
1	2 6
2	
3	1
4	2 8
5	6
6	3 4
7	8
8	3
9	1

From this, students should realize that 56 is the middle number.

This representation allows for easy identification of the number that occurs most frequently in a data set. In this data set, there is no mode. Students should also work with data sets that have one mode and those that have more than one mode. When determining mode, they should also explore data sets that are categorical in nature (e.g., favourite sport).

Data sets can be provided to students or collected through surveys. Students should develop awareness that certain data sets are better represented by one or more measures of central tendency. As they evaluate the data, they must decide which measures of central tendency best represent the average of the data.

When determining measures of central tendency, data sets should be limited to a manageable number of values and should contain both even and odd numbers of entries. Data sets can be sorted but students should also be exposed to examples where they have to first order the data before calculating the median. Students should be exposed to contextual situations in which they are required to calculate various measures of central tendency.

General Outcome: Develop statistical reasoning.

Suggested Assessment Strategies

Journal

- Ask students to describe how they would explain the difference between mean, median, and mode to a friend who missed the class when measures of central tendency were introduced. They should use an example to support the explanation.

(S1.1)

Paper and Pencil

- Samantha is a hairdresser at a busy salon in Mount Pearl, NL. Her customers often leave tips for her service. One afternoon, Samantha gave four haircuts and earned tips of $7.25, $6.50, $5.00, and $10.00. She is hoping to earn a mean amount of $7.50 in tips per haircut. If she has one more appointment scheduled for the day, ask students to determine how much she must earn from the tip to reach her goal.

(S1.2)

Resources/Notes

Authorized Resource

Math at Work 12

2.1 Measures of Central Tendency

SB: pp. 64-79

TR: pp. 54-63

Statistics

Outcomes

Students will be expected to

S1 Continued ...

Achievement Indicator:

S1.3 *Identify and correct errors in a calculation of a measure of central tendency.*

Elaborations—Strategies for Learning and Teaching

It may be helpful at this point to review with students the proper use of the calculator when determining the mean to eliminate possible calculation errors. Students may try to calculate the sum of the values in the data set and divide by the total number of data values in the same step. In this case, the answer given by the calculator may not be correct since order of operations may not have been followed. Students could be encouraged to write all steps in the solution to alleviate this potential error.

Some other possible common errors may occur when students are working with measures of central tendency:

- They may not arrange the data in ascending or descending order prior to determining the median.
- The median may be incorrectly identified in an even-numbered data set as both middle numbers or as one of the two.
- Students report cases where no mode exists as having a mode of 0.

General Outcome: Develop statistical reasoning.

Suggested Assessment Strategies

Journal

- Ask students to respond to the following:

  Julia and Marcus were both asked to determine the median of the following data set:

  5, 8, 3, 14, 21, 16, 9, 18, 4

  Marcus stated that the median was 9 and Julia said it was 21. Which student is correct? Explain your reasoning.

  (S1.3)

Performance

- Students could be divided into small groups to work through a series of station activities. At each station, students are presented with a sample solution for a calculation of a measure of central tendency. Students are asked to identify all errors that have been made in the solution, discuss why the errors have been made, and then provide a correct solution.

  Examples:

  (i) Given the following set of data, determine the mean:

  3, 4, 3, 9, 8, 7, 6, 7, 7

  Student Solution:

  $$\text{Mean} = \frac{3+4+3+9+8+7+6+7+7}{8}$$
  $$= \frac{54}{8}$$
  $$= 6.75$$

  (ii) Given the following set of data, determine the median:

  6.2, 7.4, 5.9, 6.0, 6.3, 6.9, 7.5, 7.2

  Student Solution:

  Ordered data from least to greatest ...

  5.9, 6.0, 6.2, 6.3, 6.9, 7.2, 7.4, 7.5

  Since there are 8 numbers, the median is the fourth number.

  (iii) Given the following set of data, determine the mode:

  54, 53, 56, 51, 54, 56, 58, 59

  Student Solution:

  Since both 54 and 56 occur most often, the mode is:

  $$\frac{54+56}{2} = 55$$

  (S1.3)

Resources/Notes

Authorized Resource

Math at Work 12

2.1 Measures of Central Tendency

SB: pp. 64-79

TR: pp. 54-63

Statistics

Outcomes

Students will be expected to

S1 Continued ...

Achievement Indicator:

S1.4 *Explain, using examples, the advantages and disadvantages of each measure of central tendency.*

Elaborations—Strategies for Learning and Teaching

Once students have been exposed to calculating measures of central tendency, classroom discussion should then focus upon realistic situations to explore the advantages and disadvantages of each of these measures of central tendency. Students have previously compared these three measures to determine which is most appropriate in a given situation. Discussion may focus on data sets such as test scores, shoe size, favourite music genre or heights of trees.

Sample advantages and disadvantages of the measures of central tendency are shown below. This list is not meant to be exhaustive.

Measure of Central Tendency	Advantages	Disadvantages
Mean	• commonly used in familiar contexts • easy to calculate • useful when comparing sets of data	• skewed by extreme values (outliers)
Median	• extreme values do not affect this measure as strongly as they do the mean	• tedious to arrange large sets of data in order without technology
Mode	• extreme values do not affect the mode • useful when the data values are limited in scope (e.g., shoe size) • can be used with non-numerical data sets (e.g., favourite color)	• a set of data may have no mode • there may be more than one mode which can be difficult to interpret

Although students may initially choose the mean to represent the average of a data set, this discussion should demonstrate that it may be more advantageous to use either median or mode in certain situations. They should consider the appropriateness of each measure depending upon the situation presented.

General Outcome: Develop statistical reasoning.

Suggested Assessment Strategies

Interview

- Present students with the following situations. Ask them to determine whether mean, median, or mode would be most helpful and have them justify their choice.
  (i) You are ordering bowling shoes for a bowling alley.
  (ii) You want to know if you read more or fewer books per month than most people in your class.
  (iii) You want to know the "average" amount spent per week on junk food in your class.

(S1.4)

Journal

- Ask students to respond to the following:

  Darryl, Gordon, and Joan are captains of the school math teams. Their contest results are recorded in the table below.

	Darryl	Gordon	Joan
Contest 1	82	84	85
Contest 2	82	84	85
Contest 3	88	90	85
Contest 4	100	71	81
Contest 5	77	78	81
Contest 6	81	87	85
Contest 7	87	89	82
Contest 8	83	88	85
Contest 9	83	86	83

  Which measure of central tendency would you choose to determine who has the best team? Why?

(S1.4)

Resources/Notes

Authorized Resource

Math at Work 12

2.1 Measures of Central Tendency

SB: pp. 64-79

TR: pp. 54-63

Statistics

Outcomes

Students will be expected to

S1 Continued ...

Achievement Indicators:

S1.5 *Explain, using examples such as course marks, why some data in a set would be given a greater weighting in determining the mean.*

S1.6 *Calculate the mean of a set of numbers after allowing the data to have different weightings (weighted mean).*

Elaborations—Strategies for Learning and Teaching

In certain situations the straight forward calculation of mean does not adequately represent the given set of data. Students will now be introduced to the concept of weighted mean and how it is determined. Weighted mean is a calculation that takes into account the relative importance of the individual data values. A discussion of how students' course marks are calculated will provide rationale for the use of weighted mean and why certain values in a data set are given greater weighting. If a student's overall grade, for example, is based on marks on two quizzes and one test, and the test is worth 60% of the grade while each quiz is worth 20%, it is the weighted mean that will give the student's final grade.

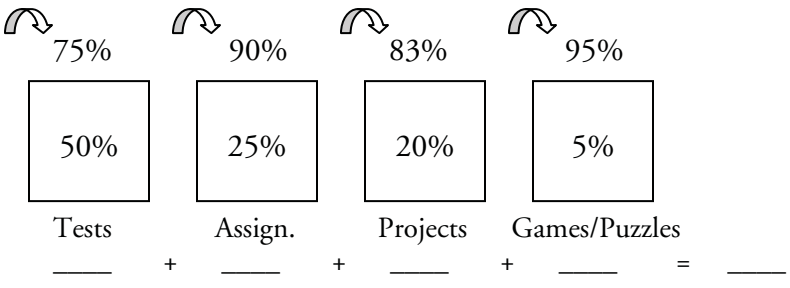
To introduce the calculation of weighted mean, students could be provided with a sample course evaluation scheme and a fictional student's marks such as in the table below:

Category	Weighting	Student's Average
Tests & Quizzes	50%	75%
Assignments	25%	90%
Projects	20%	83%
Games / Puzzles	5%	95%

Ask them to do a comparison of mean versus the weighted mean for the above data. This can lead to a discussion of how a student's average is obtained in each category compared to their overall average.

After analyzing and calculating weighted mean for various sets of data, students should realize the difference between mean and weighted mean and be able to identify an appropriate context for each.

A graphic organizer, such as the following, could be useful for this activity.



General Outcome: Develop statistical reasoning.

Suggested Assessment Strategies

Journal

- A student was given the following grading scheme for a course:

  Tests & Quizzes – 30%

  Assignments – 20%

  Projects – 20%

  Final Exam – 15%

  Homework – 10%

  Games / Puzzles – 5%

  Ask students to discuss whether they would want the mean of all work completed to represent their final grade in this course.

  (S1.5)

- Ask students to respond to the following:

  Two candidates are applying for the same job. The following weightings were assigned:

  - Presentation - 40%
  - Interview Questions - 25%
  - Written Essay - 35%

  Steven scored 85% on his interview, 75% on his presentation, and 70% on his written essay.

  Susan scored 90% on her interview, 60% on her presentation, and 50% on her essay.

  Based on these results, who will be hired?

  If you were competing for this job, which of the three scroing categories do you think would be your strength?

  (S1.6)

Interview

- Emily had an overall average of 67% in a math course. On her next assignment she received a grade of 95%. Ask students how Emily could explain to her parents the reason her overall average did not change significantly as a result of the most recent assignment.

  (S1.5)

Paper and Pencil

- Ask students to answer the following:

  Jean-Paul teaches French classes at a community college in Corner Brook, NL. He gave two tests, one worth 60% of the final mark and the other worth 40% of the final mark. If a student earned 80% on the first test and 70% on the second test, what would be his final mark? (S1.6)

Resources/Notes

Authorized Resource

Math at Work 12

2.1 Measures of Central Tendency

SB: pp. 64-79

TR: pp. 54-63

Statistics

Outcomes

Students will be expected to

S1 Continued ...

Achievement Indicators:

S1.7 *Identify the outlier(s) in a set of data.*

S1.8 *Explain the effect of outliers on mean, median and mode.*

S1.9 *Calculate the trimmed mean for a set of data, and justify the removal of the outliers.*

Elaborations—Strategies for Learning and Teaching

A set of data often contains outliers, or values which are significantly different from the others. The presence of outliers may affect which measure of central tendency best represents the data. Students have been previously exposed to this idea but it will be necessary to revisit it. They should be able to identify the outlier(s) and explain the effect upon measures of central tendency.

In some cases, the presence of outliers may not affect the measures of central tendency. Ask students to explore the effect of 38 and 98 on the measures of central tendency for the data set {38, 64, 68, 68, 71, 72, 75, 98}. In this case, they should conclude that the values on opposite extremes of the data set will have virtually no effect on the average score.

Students should also analyze cases where there is only one outlier or multiple outliers on the same extreme. Sometimes the median can be affected, as it would be in data sets such as {1, 2, 4, 6, 63} and {3, 5, 26, 33, 37, 42}.

Once students examine sets of data containing outliers, they should realize that the presence of these outliers can misrepresent the "middle" of a set of data. If one is studying the average temperature of objects in a kitchen, for example, most would be at room temperature, between 20°C and 25°C. If a warm oven at 300°C is included as one of these objects, the median would be close to room temperature, but the mean would be much higher. For this situation, the median would be the better choice.

The concept of trimmed mean is new to students. Trimmed mean is a calculation of the mean after removing the least and greatest value(s) in a data set. This is done to obtain a better representation of the central tendency of the data. In order to calculate trimmed mean, the same number of values should be removed from each end of an ordered list and the mean should be calculated from the remaining data values. These discarded values are not always outliers although they often can be. A trimmed mean will alleviate the misrepresentation of data caused by outliers.

Trimmed mean is used to score Olympic events such as gymnastics to eliminate potential bias (outliers) from judges. The highest and lowest judges' scores are eliminated and the trimmed mean is calculated with the remaining scores.

General Outcome: Develop statistical reasoning.

Suggested Assessment Strategies

Performance

- Fill three identical containers with approximately 15 different-sized objects (e.g., paper clips, pencils). Each container should have one object significantly smaller and significantly larger than the rest. Students are divided into groups and given a container. They should measure and record the length of each item, arrange their objects from shortest to longest, and identify objects that would be considered outliers. Ask them to calculate the mean length, eliminate the outliers, and calculate the trimmed mean. As a class, compare the arithmetic means and trimmed means and discuss any differences that are found.

  (S1.7, S1.8, S1.9)

- Students can collect data about the length of time it takes for each class member to run 100 m. They can then calculate the mean and trimmed mean and discuss whether it may be more appropriate to use a trimmed mean. Brainstorm examples where a trimmed mean may be more representative of the average time taken to run 100 m. (S1.9)

Paper and Pencil

- Ask students to answer the following:

  Tanya received the following scores on her first five science quizzes:

  75%, 75%, 80%, 77%, 82%

  On her next quiz, she only achieved a mark of 52%. What effect, if any, did this mark have on the measures of central tendency?

  (S1.8)

- During the last six months, Steve, a real estate agent, sold 9 houses at the following prices:

$1 479 000	$750 000	$699 000
$435 900	$659 000	$589 500
$449 900	$625 600	$712 800

  Ask students to:

  (i) Calculate the mean and the trimmed mean.
  (ii) Which would be a better indication of the average price of a house sold? Why?

  (S1.2, S1.8, S1.9)

Journal

- The average annual household income reported in a community is $85 000. Last week, one community member won $5 million in the lottery. Ask students to discuss if this will affect the average annual household income next year. (S1.8)

Resources/Notes

Authorized Resource

Math at Work 12

2.2 Using Other Statistical Measures

SB: pp. 80-93

TR: pp. 64-70

Statistics

Outcomes

Students will be expected to

S1 Continued ...

Achievement Indicator:

S1.10 *Explain, using examples from print and other media, how measures of central tendency and outliers are used to provide different interpretations of data.*

Elaborations—Strategies for Learning and Teaching

Measures of central tendency are frequently cited in print and other forms of media. Students should be aware that the choice of reporting mean, median or mode is consciously made in order to emphasize a particular perspective. Even when a reported measure of central tendency is accurate, it may not always provide a true representation of the data being discussed. Students should use their knowledge of these measures of central tendency to interpret data presented in the media. Reports of statistics in the media are often made with a particular audience in mind. Often, the median is reported for data that may be skewed one way or the other. An article on NHL player salaries, for example, may report the median salary. This information would give no indication of the salaries for some of the highest paid players. On the other hand, the mean salary would indicate an overinflated answer to the question, "How much does an NHL player make?"

Another example that can be considered here is the reported average housing prices for a particular area during a given month. The decision to report the mean instead of the median price can create a completely different perspective. In the case where a number of expensive houses were sold in a given time frame, the mean price can indicate that the average house in that area is more expensive than it actually is.

Students should be encouraged to analyze the reasons why particular measures of central tendency may be reported and the change in perspective that would occur if a different measure was selected.

General Outcome: Develop statistical reasoning.

Suggested Assessment Strategies

Presentation

- Students could research an article of interest that reports a measure of central tendency. They may consider such topics as car sales, salaries of actors, wages in a province, sports statistics, etc. Students can display their findings using a poster, PowerPoint, etc. and present to the class.

(S1.10)

Performance

- Students could research Sidney Crosby and his hockey career, with a focus on the following:
  (i) number of points achieved per year
  (ii) outliers that exist
  (iii) causes of these outliers

(S1.10)

- Ask students to choose a topic to research with the goal of creating an advertisement. For the ad, they should select a measure of central tendency that would skew the interpretation of the data.

(S1.10)

- Given a set of flyers (e.g., food, cars, furniture), ask small groups of students to create their own media spots. They should choose the measure of central tendency which will make the most convincing case to sell the product to the consumer.

(S1.10)

- Students could collect a series of tv ads / print ads relating to claims based on central tendencies. Examples could include:
  (i) average increase in hair growth
  (ii) average weight loss on a program
  (iii) average savings on a cartload of groceries
  (iv) average savings when buying a car outside of the urban area
  (v) how much, on average, can be saved by using a mortgage broker

  Ask students to rewrite one of the ads using a different measure of central tendency to make a different claim.

(S1.10)

Resources/Notes

Authorized Resource

Math at Work 12

2.2 Using Other Statistical Measures

SB: pp. 80-93

TR: pp. 64-70

Statistics

Outcomes

Students will be expected to

S2 Analyze and describe percentiles.

[C, CN, PS, R]

Achievement Indicators:

S2.1 *Explain the relationship between median and percentile.*

S2.2 *Explain, using examples, percentile ranks in a context.*

S2.3 *Explain, using examples, the difference between percent and percentile rank.*

Elaborations—Strategies for Learning and Teaching

Students are formally introduced to the concept of percentile and percentile rank. The link between percentile and median will be made. Percentiles are often used to organize sets of data, such as test scores or birth weight. Percentile ranks compare individual values with the other values in a data set. Students have previously worked with the concept of percent and its applications. This experience will be used to distinguish between appropriate uses of percentile ranks compared to percents.

Percentiles divide a distribution of data into two or more groups. If a teacher wishes to determine the exam score that divides the class in half, for example, the point that marks the 50^{th} percentile is determined. The 50^{th} percentile corresponds to the the median value in a data set. Students will also have to work with the 25^{th} and 75^{th} percentiles, representing the medians of the lower and upper halves of the data set respectively. The terms lower and upper quartile could be introduced.

A percentile rank is used to determine where a particular score or value fits within a broader distribution. It shows the percentage of values in a set that are at or below a given value. To compare a score of 8 with the rest of the scores on a quiz marked out of 10, a percentile rank would be useful:

6, 6, 6, 7, 7, 7, 7, 8, 8, 10

Since 9 of the 10 scores are at or less than 8, the percentile rank for that score is 90^{th} rank.

Students can use the following formula to determine percentile ranking:

$$PR = \frac{\text{number of values at or below the value being considered}}{\text{total number of data values}} \times 100$$

They should understand that the percentile rank is not the percent score earned by an individual student. An example such as the following highlights this difference.

Ask students to consider the following quiz scores:

75, 77, 78, 78, 80, 81, 81, 82, 83, 84, 84, 85, 87, 87, 88, 88, 88, 89, 90, 92

A score of 80% initially sounds good. However, this doesn't sound as impressive when it is thought of as the 25^{th} percentile rank, or only 25% of the class scored 80% or less.

It is a good idea to begin work percentile ranks with data values that are not percents, since some students may struggle to understand the difference between percent and percentile rank.

General Outcome: Develop statistical reasoning.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to determine the 50th percentile for each data set:
  (i) 62, 45, 39, 82, 51, 29, 66
  (ii) 3, 8, 12, 15, 4, 22, 13, 20

(S2.1)

Interview

- Ask students what conclusions can be made about a test where a mark of 50% is in the 80th percentile.

(S2.2, S2.3)

Journal

- Ask students to respond to the following:
  (i) On a recent test, Laura received a score of 85%. This placed her in the 96th percentile of students who wrote the test. Explain how Laura did on the test compared to her classmates.

(S2.2, S2.3)

  (ii) The following data shows the amount of time (in minutes) a student spends on Math homework during one week: 20, 0, 15, 30, 10. Determine the median and explain what percentile this represents.

(S2.1)

Resources/Notes

Authorized Resource

Math at Work 12

2.2 Using Other Statistical Measures

SB: pp. 80-93

TR: pp. 64-70

Algebra

Outcomes

Students will be expected to

A1 Demonstrate an understanding of linear relations by:

- **recognizing patterns and trends**
- graphing
- creating tables of values
- writing equations
- **interpolating and extrapolating**
- solving problems.

[CN, PS, R, T, V]

Achievement Indicator:

A1.1 *Describe the trends in the graph of a data set, including scatter plots.*

Elaborations—Strategies for Learning and Teaching

Students have worked with linear relations in the intermediate grades. They used algebraic expressions to describe patterns, constructed graphs from the corresponding table of values, and examined the various ways a relation can be expressed, including ordered pairs, table of values and graphs. They also used patterns to find missing values in a linear relation (7PR2, 7PR4, 8PR1). In Grade 9, students generalized patterns arising from a problem-solving context (9PR1) and graphed, analyzed, interpolated and extrapolated to solve problems (9PR2). In Mathematics 2202, they interpreted graphs by describing trends and interpolating and extrapolating values (S1). These topics will be further explored in this unit as their relationship to scatter plots is developed.

A scatter plot is a graph used to determine the type of relationship that exists (if any) between two variables. Scatter plots are especially useful with large quantities of data because they help in the visualization of trends. The focus here is on linear trends. Students should be able to identify that as the independent variable increases the dependent variable will increase, decrease, or show no trend. They have experience describing trends in the graph of a given data set from Mathematics 2202.

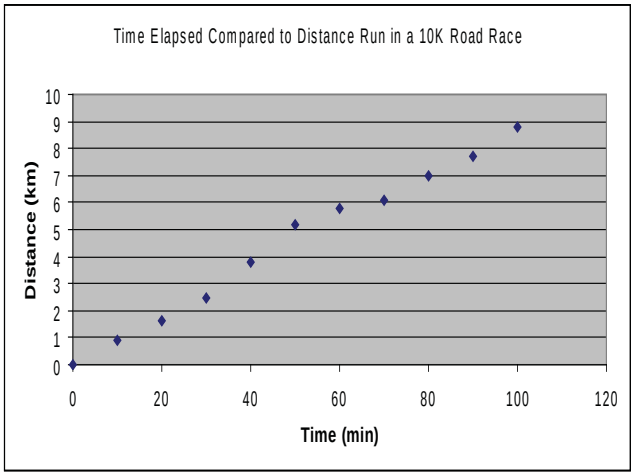
The intent of this outcome is to examine and generalize trends in scatter plots. It is not necessary to introduce the notions of correlation or lines of best fit at this time. These concepts will be developed later in this course.

General Outcome: Develop algebraic reasoning.

Suggested Assessment Strategies

Paper and Pencil

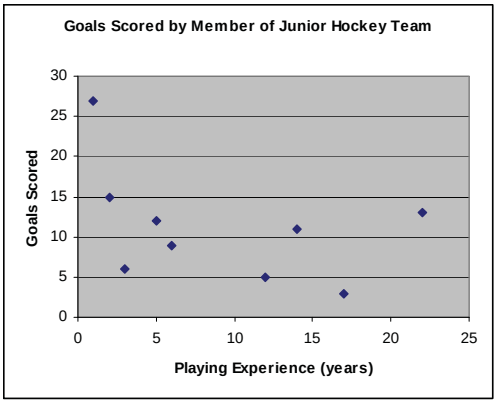
- The following scatter plot shows the distance covered by a runner in a 10 km road race:



  Ask students if the relationship between distance and time appears to be linear or not. They should provide an explanation.

  (A1.1)

- Ask students to describe the trend in the scatter plot:



  (A1.1)

Resources/Notes

Authorized Resource

Math at Work 12

2.3 Scatter Plots

SB: pp. 94-101

TR: pp. 71-75

Algebra

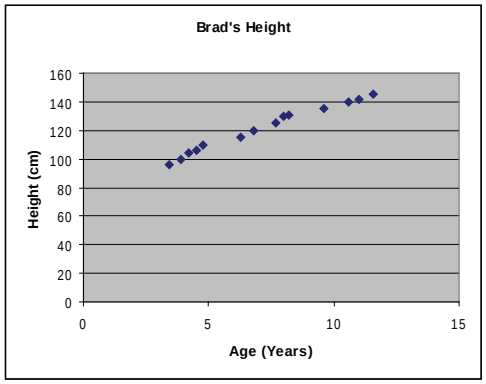
Outcomes	Elaborations—Strategies for Learning and Teaching
Students will be expected to **A1 Continued ...**	
Achievement Indicator: A1.2 *Solve a contextual problem that requires interpolation or extrapolation of information.*	The focus here will be on interpreting the data and making predictions for unknown values. Interpolation is the prediction of a value between two known values. Extrapolation is the prediction of a value which goes beyond the data that is given. In Mathematics 2202, students interpolated and extrapolated values from a given graph. There is opportunity here for practical applications. By extending the graph, assumptions are being made that the pattern will continue. Students need to be aware that this is not always applicable in a given context. Discuss, for example, a scatter plot comparing the heights of individuals and their age. This graph would show a positive trend up to a certain age. Students should recognize that this trend does not continue since growth does not continue throughout a person's life span. It is important, therefore, that students justify their interpolations and extrapolations as they make inferences from a graph.
A1.3 *Match given contexts with their corresponding graphs, and explain the reasoning.*	Students should be able to match a scatter plot with a given trend as well as provide examples of contextual situations that could be represented by a given scatter plot. Ask them to discuss situations for each of the following: • the scatter plot does not show any trend • as one variable increases, so does the other • a decreasing trend that cannot continue indefinitely.
N1 Analyze puzzles and games that involve logical reasoning, using problem-solving strategies. **[C, CN, PS, R]**	The puzzles and games outcome could be incorporated into a review of topics introduced in this unit. A game could be modelled after the reality television game show, The Amazing Race™. Students work in teams of two to complete the race. In each leg of the race, teams answer a question based on the measures of central tendency, percentiles, scatter plots, or interpolation and extrapolation. When they correctly answer the question, a clue is provided that leads the team to the next destination. The team that arrives first in the final leg wins the game.

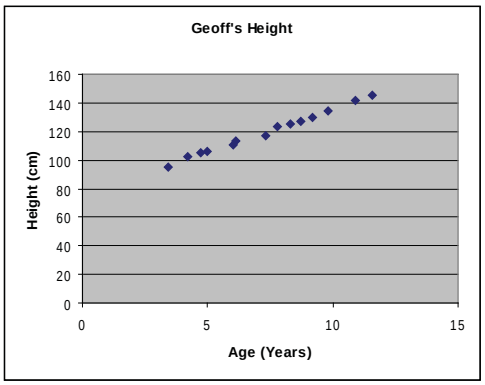
General Outcome: Develop algebraic reasoning.

Suggested Assessment Strategies

Paper and Pencil

- Brad and Geoff are brothers. Their parents track their growth on the following growth charts:





Ask students to answer the following:

(i) Based on current growth trends, who will be taller at age 12, Brad or Geoff?

(ii) Could the given chart be used to predict Brad's height at age 45 years? Explain your reasoning.

(A1.2)

- Present small groups of students with a scatter plot of mystery data. Students should hypothesize what is represented by each. Assistance may be offered by giving a list of the data sets represented and allowing groups to select which is depicted by their example.

(A1.3)

Performance

- Students can create a foldable to display scatterplots that are increasing, decreasing, and showing no trends. On the outside panels, they can describe the trend. On the corresponding inside panel, they can draw a sketch of the scatter plot and give an example of a situation that could be represented by this trend. (A1.3)

Resources/Notes

Authorized Resource

Math at Work 12

2.3 Scatter Plots

SB: pp. 94-101

TR: pp. 71-75

Games and Puzzles

Penny Ball

SB: p. 107

TR: p. 82

Linear Relationships

Suggested Time: 18 Hours

Unit Overview

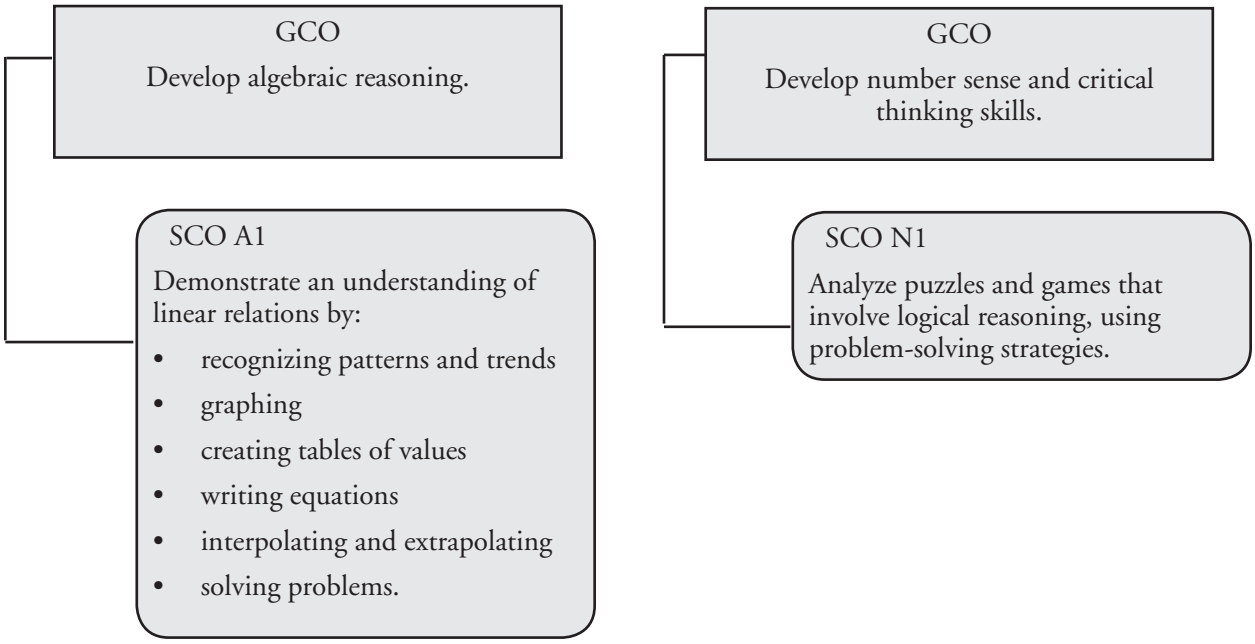
Focus and Context

This unit continues the work with scatter plots that was introduced in the Working With Data unit. Linear and nonlinear trends within scatter plots are analyzed. The relation between the variables is represented with a trend line, or line of best fit.

Students represent relations using tables of values and graphs. They analyze the table to determine if there are any patterns in the data. Graphs are then used to visualize and display patterns. They see that a linear relation represented on a graph can pass through the origin in direct variation or pass through another point on the *y*-axis in partial variation.

This unit builds on the concepts of slope and linear relations from Mathematics 2202 to develop equations of lines as a way to describe relationships among data.

Outcomes Framework



Mathematical Processes

[C] Communication
[CN] Connections
[ME] Mathematical Modeling and Estimation
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

SCO Continuum

Mathematics 1202	Mathematics 2202	Mathematics 3202
Algebra		
not addressed	S1. Solve problems that involve creating and interpreting graphs, including: • bar graphs • histograms • line graphs • circle graphs. [C, CN, PS, R, T, V] A2. Demonstrate an understanding of slope: • as rise over run • as rate of change • by solving problems. [C, CN, PS, V]	A1. Demonstrate an understanding of linear relations by: • recognizing patterns and trends • graphing • creating tables of values • writing equations • interpolating and extrapolating • solving problems. [CN, PS, R, T, V]
Number		
G1. Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies. [C, CN, PS, R]	N1. Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies. [C, CN, PS, R]	N1. Analyze puzzles and games that involve logical reasoning, using problem-solving strategies. [C, CN, PS, R]

Algebra

Outcomes

Students will be expected to

A1 Demonstrate an understanding of linear relations by:

- **recognizing patterns and trends**
- **graphing**
- **creating tables of values**
- **writing equations**
- **interpolating and extrapolating**
- **solving problems.**

[CN, PS, R, T, V]

Elaborations—Strategies for Learning and Teaching

In the Working With Data unit, students described the trends in a given scatter plot. They will now create scatter plots. They will identify linear and nonlinear relationships, and model direct variation and partial variation relationships with tables of values, graphs and equations. Students will also solve problems involving linear trends and direct and partial variation relationships.

Achievement Indicators:

A1.4 *Create, with or without technology, a graph to represent a given or collected data set.*

Using given or collected data, students should create scatter plots. It will be necessary to review graphing skills such as plotting points, labelling axes and choosing an appropriate scale. Although the terms independent and dependent variables are not new to students, it is important to reiterate that the independent variable is graphed along the horizontal axis while the dependent variable is represented on the vertical axis. By drawing a line of best fit students will be able to determine linear or nonlinear trends.

Technology, such as graphing calculators, FX Draw and spreadsheets, could be used to create graphs. The intent is not for students to become proficient at using various graphing software. Technology is meant to provide them with a quick and accurate visual to identify trends.

A1.2 *Solve a contextual problem that requires interpolation or extrapolation of information.*

Students use the line of best fit to extrapolate and interpolate information. Caution them about making assumptions on the continuation of a pattern. A graph may show a positive increase but this trend may not continue in a contextual situation. Discuss, for example, a scatter plot comparing the time a weight lifter spends training to the amount she can bench press. The graph would show a positive trend up to a certain weight. Sports statistics and Olympic records can be used to illustrate that some linear trends will cease at some point.

Interpolating is likely to be more accurate since it is between two known values. Extrapolating, on the other hand, is less reliable because a new trend could occur. When constructing a scatter plot, students may not leave enough space to extrapolate. Encourage them to read an entire question before drawing a graph in case they need to extrapolate.

General Outcome: Develop algebraic reasoning.

Suggested Assessment Strategies

Paper and Pencil

- On the Robin family road trip, the distance (km) over time (h) is given in the table. Ask students to graph the following data and draw a line of best fit.

Time (h)	Distance (km)
0	0
1	95
2	210
3	305
4	420
5	525

They should use the graph to answer the following:

(i) How far were they after 250 minutes?
(ii) How far will they be after 9 hours?

(A1.2, A1.4)

- For her last four science tests, Sarah recorded her study time and the resulting grades.

Study Time (h)	Test Grade
0	41%
1	52%
2	60%
3	71%

Ask students to graph the data and draw a line of best fit. They should use the table or graph to answer the following:

(i) Approximately what grade should she expect on her next test if she studies for 2.5 hours?
(ii) Approximately what grade should she expect on her next test if she studies for 4 hours?
(iii) Can Sarah expect this trend to continue? Explain.

(A1.2, A1.4)

Resources/Notes

Authorized Resource

Math at Work 12

3.1 Understanding Linear Trends and Relationships

Student Book (SB): pp. 112-126

Teacher's Resource (TR): pp. 90-98

Algebra

Outcomes	Elaborations—Strategies for Learning and Teaching
Students will be expected to **A1 Continued ...** **Achievement Indicators:**	
A1.5 *Sort a set of scatter plots according to the trends represented (linear, nonlinear or no trend).*	When working with scatter plots and lines of best fit, students saw graphs with linear and nonlinear trends. They should be exposed to a variety of graphs and asked to sort them according to these trends. Students should apply the trends they discover to contextual situations. For given graphs students could be asked to discuss questions such as: • Does the scatter plot show a trend? • Is the trend linear or nonlinear? Explain how you know. They should realize that a random scattering of data points indicates no relation between the variables, while a clustering with a general tendency indicates a relation. They should also note that how closely the data points are clustered along a line of best fit indicates how close the relation is.
A1.3 *Match given contexts with their corresponding graphs, and explain the reasoning.*	Students should match a given context with a graph, as well as provide examples of contextual situations that could be represented by the given graph. When matching graphs with contextual situations, they should identify the independent and dependent variable. This provides an opportunity to review these concepts with students.
A1.6 *Identify and describe the characteristics of a linear relation represented in a graph, table of values or number pattern.*	Students have used a table of values to construct a scatter plot and identified linear and nonlinear trends in a graph. The relationship between linear and nonlinear trends and the rate of change given a table of values should also be investigated. They should realize that a constant rate of change in the table indicates a linear relationship.
A1.7 *Sort a set of graphs, tables of values and/or number patterns into linear and nonlinear relations.*	Once students recognize the characteristics of a linear relation, they should sort sets of graphs, tables of values and/or number patterns into linear and nonlinear relations and justify how they sorted the sets. Consider an activity where students are given linear and nonlinear relations in the various formats. Ask students such questions as: • Why are the tables linear? • What makes the graph nonlinear?

General Outcome: Develop algebraic reasoning.

Suggested Assessment Strategies

Interview

- Ask the student to describe situations where the data would represent a linear trend, a nonlinear trend, and no trend.

(A1.3)

Performance

- Provide students with various graphs and tables. Ask them to sort them into linear and nonlinear relations.

(A1.5)

- Students could create a three tab foldable. On the outside of each of the tabs they could sketch three scatter plots: linear, nonlinear and no trend. Characteristics of each trend could be listed on the inside.

(A1.5)

- A memory matching game could be played in small groups. Students have to find matching cards: one of the table of values and one of the graph. If they can correctly identify the relationship as linear or nonlinear, they keep the matching pair.

(A1.6, A1.7)

- Students could participate in a *Think-Pair-Share* activity.

  Step 1: Think

  Individually, students brainstorm scenarios where linear relationships exist.

  Step 2: Pair

  Students link up with a partner to discuss ideas.

  Step 3: Share

  The class compiles all the ideas.

(A1.6)

Journal

- Given a choice of different graphs, ask students to write a story describing the relationship between the variables. It could be humorous, action-filled, tragic, child-friendly, etc.

(A1.3)

Resources/Notes

Authorized Resource

Math at Work 12

3.1 Understanding Linear Trends and Relationships

SB: pp. 112-126

TR: pp. 90-98

Algebra

Outcomes

Students will be expected to

A1 Continued ...

Achievement Indicator:

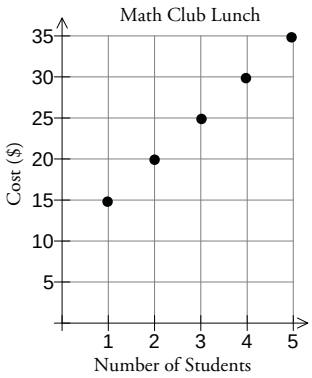
A1.8 *Explain why the points should or should not be connected on the graph for a context.*

Elaborations—Strategies for Learning and Teaching

Although students worked with discrete and continuous data in Grade 9 (9PR2), a review of these concepts is necessary. Discrete data is data which can be counted so it does not contain fractions. When graphing data points that represent discrete data, points are not connected or are connected with a dashed line. A dashed line is used when the discrete data have values between the plotted points that are valid. If there are no valid values between the plotted points, then no line is drawn. Continuous data has an infinite number of values between data points. It makes sense to have fractions. When graphing points that represent continuous data, points are connected with a solid line.

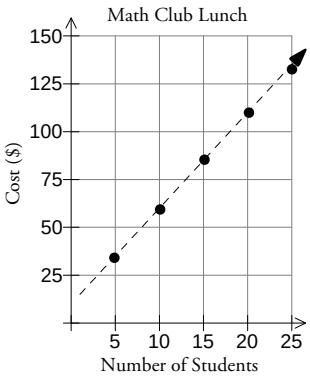
Contextual situations such as the following should make this idea more concrete for students:

- Case 1: No line drawn



This graph has discrete data because it is not possible to have a fraction of a person. Since there are no valid data points between the plotted points, the points are not connected.

- Case 2: Dashed line is drawn



This graph also contains discrete data. Since there are some valid data points between the plotted points, the points are connected with a dashed line. Three students would cost $25, for example, so the point (3, 25) is a valid point.

General Outcome: Develop algebraic reasoning.

Suggested Assessment Strategies

Journal

- Ask students to determine if the data point should or should not be connected in the following situations.
  (i) The cost of 150 people renting a banquet hall at $6.50 per person
  (ii) The cost of 10 students going to a movie if tickets cost $5 person
  (iii) The amount of gas consumed after 500 km
  (iv) The number of song downloads in iTunes:

Day	1	2	3	4	5
# of Downloads	6	9	12	15	18

(A1.8)

Resources/Notes

Authorized Resource

Math at Work 12

3.2 Direct Variation

SB: pp. 127-142

TR: pp. 99-110

3.3 Partial Variation

SB: pp. 143-159

TR: pp. 111-122

Algebra

Outcomes

Students will be expected to

A1 Continued ...

Achievement Indicators:

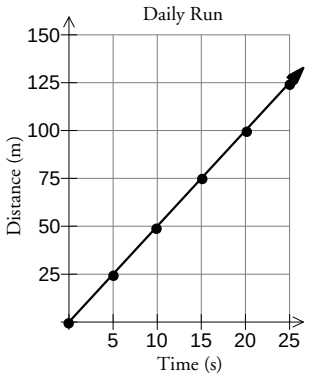
A1.8 *Continued*

A1.9 *Relate slope and rate of change to linear relations.*

A1.10 *Identify a linear equation as having a direct or partial variation relationship.*

Elaborations—Strategies for Learning and Teaching

- Case 3: Solid line is drawn



On this graph the data is continuous because it makes sense to have fractional time. The points, therefore, are connected with a solid line.

In Mathematics 2202, students worked extensively with slope. They developed slope in terms of rise over run and explained slope as rate of change using examples and illustrations (A2). Students should now compare tables and graphs to make the connection between slope and rate of change. They should recognize that the constant change in the independent variable in the table of values represents the horizontal change in the graph. Similarly, the constant change in the dependent variable represents the vertical change in the graph. The rate of change in the table, therefore, represents the slope of the graph.

A direct variation is a linear relationship in which one variable is always a fixed multiple of the other variable. A partial variation is a linear relationship in which one variable is always a fixed multiple of the other variable plus a constant amount. It is an indication that a portion of the dependent variable is predetermined and does not change as the independent variable changes. Discuss taxi cab fare as an example of partial variation. Usually, a taxi fare begins with a flat fee for entering the taxi, and then the charge is a relationship between distance travelled and time. The graph of a direct variation relationship starts at the origin, resulting in a y-intercept of zero. The graph of a partial variation relationship does not start at the origin. Students could examine this using the equations $y = 4x$ and $y = 4x + 5$. They should see that the first equation will represent a relationship that has a direct variation as the y-value will be multiples of four. The second equation represents a relationship with a partial variation as the y-value will be multiples of four plus five. The numbers will not be multiples of four, but they will increase by four. Students should make the connection between the fixed multiple in the equation and the slope of the graph. They should also realize that the constant amount added is the y-intercept.

General Outcome: Develop algebraic reasoning.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to answer the questions that follow based on the data provided on a movie streaming service for subscribers.

# of Movies Streamed	Cost ($)
0	8.00
1	10.00
2	12.00
3	14.00
4	16.00

(i) Is this relation linear or nonlinear?
(ii) What is the rate of change for the relation?
(iii) What is the slope of the relation and what does it represent?
(iv) Does the relation have direct or partial variation?

(A1.6, A1.9, A1.10)

Journal

- Ask students to respond to the following:

  Cara is starting a summer lawn care service. She is debating between whether to charge $10.00 per lawn plus $5.00 for each hour it takes to complete or $25.00 per lawn.

  Which setup is most beneficial to her? Use the terms direct or partial variation in your response.

(A1.10)

Resources/Notes

Authorized Resource

Math at Work 12

3.2 Direct Variation

SB: pp. 127-142

TR: pp. 99-110

3.3 Partial Variation

SB: pp. 143-159

TR: pp. 111-122

Algebra

Outcomes	Elaborations—Strategies for Learning and Teaching
Students will be expected to **A1 Continued ...**	
Achievement Indicators: A1.11 *Create a table of values for a given equation of a linear relation.*	Students have already used tables of values when they created graphs with collected or given data. The focus here is to have them create a table of values using a given linear equation. Initially they should be given values of the independent variable to use in the equation. Students should also have the opportunity to explore situations where they choose values for the independent variable. This would lead to a discussion about which values are appropriate when creating a table.
A1.12 *Solve a contextual problem that involves the application of a formula for a linear relation.*	Students should use equations and formulas to set up tables of values to solve contextual problems. Initially the equation associated with a contextual situation is provided. Discuss how the given equation represents the situation, asking students to consider: • What does the number in front of the variable represent in the situation? (fixed multiple) In the graph? (slope) • What does the number being added represent in the situation? (constant) In the graph? (*y*-intercept) Students then apply these concepts to determine equations to represent given contexts.
A1.13 *Write an equation for a given context, including direct or partial variation.*	
N1 Analyze puzzles and games that involve logical reasoning, using problem-solving strategies. **[C, CN, PS, R]**	Revisit the puzzles and games outcome, focusing on the strategies students are using. Refer back to pp. 36-47 for additional information.

General Outcome: Develop algebraic reasoning.

Suggested Assessment Strategies

Paper and Pencil

- Sally is looking to rent a Seadoo. There are two companies in the area. The cost of a rental from Company A is $40 plus $1 per minute. Company B charges $100 per hour. Ask students to answer the following for each company:
  (i) Write an equation.
  (ii) Construct a table of values using the equation.
  (iii) Is it direct or partial variation?

  (A1.10, A1.11, A1.13)

Performance

- Students could work in pairs to construct adjoining toothpick squares, as shown below, and complete the following table of values:

Square	# of Toothpicks
1	4
2	7
3	10
4	?
5	?
6	?
7	?

  (i) Write a linear equation to represent this situation.
  (ii) Is the relationship a direct or partial variation?
  (iii) How many toothpicks would be needed to make 10 adjoining toothpick squares?

  (A1.10, A1.12, A1.13)

- Students could create an advertisement for the community ad channel. They may design an ad for a car, ATV, clothing, etc. The cost of the ad is $20.00 plus $0.25 per word. This includes one photo. Ask them to:
  (i) Create an ad.
  (ii) Write an equation to represent the relationship between the number of words and the cost.
  (iii) Is it a direct or partial variation relationship?
  (iv) Calculate the cost of the ad.

  (A1.10, A1.12, A1.13)

Resources/Notes

Authorized Resource

Math at Work 12

3.2 Direct Variation

SB: pp. 127-142

TR: pp. 99-110

3.3 Partial Variation

SB: pp. 143-159

TR: pp. 111-122

Games and Puzzles

Sudoku

SB: p. 165

TR: p. 128

BLM: 3-8

Financial Decisions

Suggested Time: 22 Hours

Unit Overview

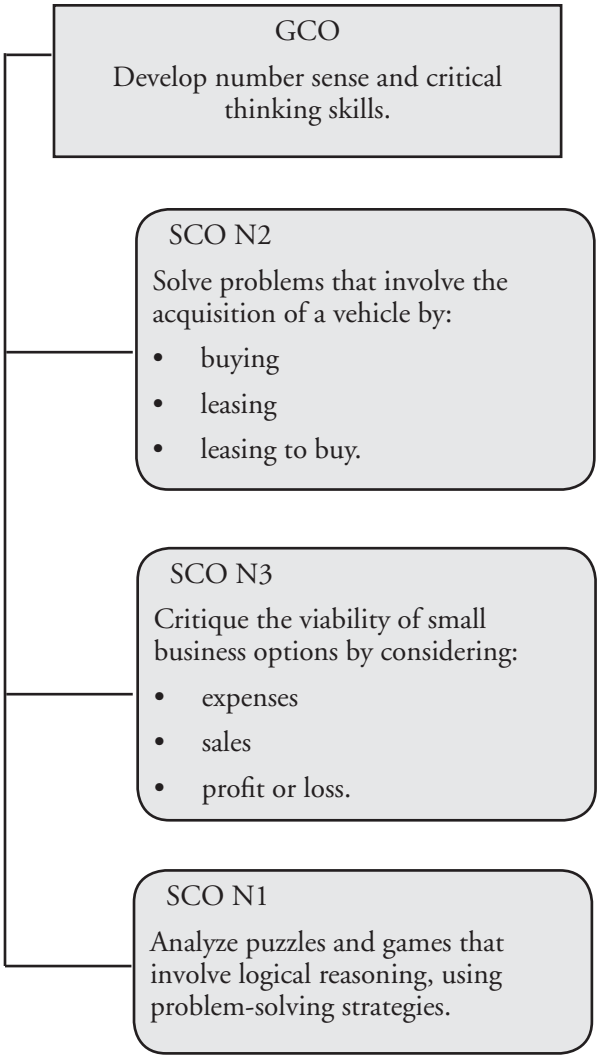
Focus and Context

In this unit, the mathematics involved in owning a vehicle and running a business is explored.

Students solve problems that involve the acquisition of a vehicle by buying, leasing, and leasing to buy. The cost, advantages and disadvantages of buying a new car and buying a used car are compared. The cost of buying a new car and leasing the same car is also compared. Students also explore some of the costs of operating a vehicle. Both fixed and variable expenses are considered.

Students analyze, interpret, and solve problems that involve critiquing the viability of small business options. They calculate annual and monthly revenue and expenses to determine whether a business has experienced a profit, loss, or break-even point.

Outcomes Framework



Mathematical Processes

[C] Communication
[CN] Connections
[ME] Mathematical Modeling and Estimation
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

SCO Continuum

Mathematics 1202	Mathematics 2202	Mathematics 3202
Number		
A1. Solve problems that require the manipulation and application of formulas related to: • perimeter • area • the Pythagorean theorem • primary trigonometric ratios • income. [C, CN, ME, PS, R] G1. Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies. [C, CN, PS, R]	N2. Solve problems that involve personal budgets. [CN, PS, R, T] N3. Demonstrate an understanding of compound interest. [CN, ME, PS, T] N1. Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies. [C, CN, PS, R]	N2. Solve problems that involve the acquisition of a vehicle by: • buying • leasing • leasing to buy. [C, CN, PS, R, T] N3. Critique the viability of small business options by considering: • expenses • sales • profit or loss. [C, CN, R] N1. Analyze puzzles and games that involve logical reasoning, using problem-solving strategies. [C, CN, PS, R]

Number

Outcomes

Students will be expected to

N2 Solve problems that involve the acquisition of a vehicle by:

- **buying**
- **leasing**
- **leasing to buy.**

[C, CN, PS, R, T]

Achievement Indicators:

N2.1 *Describe and explain various options for buying, leasing and leasing to buy a vehicle.*

N2.2 *Justify a decision related to buying, leasing or leasing to buy a vehicle, based on factors such as personal finances, intended use, maintenance, warranties, mileage and insurance.*

Elaborations—Strategies for Learning and Teaching

In the near future, students will be required to make real-life financial decisions. Given the variety of options available when acquiring a vehicle, it will be important to consider many factors when making this decision.

Students were exposed to ratios, percents and decimals in Grade 7 (7N3), Grade 8 (8N3, 8N4, 8N5) and Mathematics 1202 (A1). They explored the concept of compound interest in Mathematics 2202 (N3). They will now use these skills as they analyze and solve problems that involve acquiring a vehicle through buying, leasing, or leasing to buy.

Although most people would like to own a vehicle, students need to be aware of the options for acquiring a vehicle. Students should explain the difference between buying, leasing, and leasing to buy a vehicle. They should also identify the benefits and disadvantages of each and use this information, combined with their own personal factors, to make an informed decision. The affordability of each option should be considered. Students should explore insurance coverage and costs, such as comprehensive, collision and liability. They should recognize that new cars cost more to insure than used vehicles; leased cars require comprehensive insurance; insurance for males is higher than for females, etc. Students should also consider mileage allowances, warranty and maintenance obligations. They could explore the effect of depreciation on the value of a vehicle.

As students explore the option of buying a vehicle, they should consider options such as:

- pay the full purchase price plus sales tax
- make a down payment and take out a loan for the remainder
- take advantage of special offers from dealerships, such as reduced interest rates or clearance prices on particular models.

When considering the possibility of leasing a vehicle, they should think about:

- lease term
- lease rate
- security deposit
- kilometre allowance
- delivery charge
- option to purchase.

In addition, when considering leasing to buy, students should think about:

- residual value
- maintenace of an older vehicle.

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies

Presentation

- Ask students to collect information regarding the acquisition of a vehicle:
  (i) Identify a new or used car that you would like to own.
  (ii) Determine if you would purchase, lease, or lease to buy this vehicle.
  (iii) Using a free online insurance quote website, determine the insurance premium that you would have to pay to drive this vehicle.
  (iv) Compare your insurance premium with a partner. What factors account for the differences in insurance costs?
  Follow-up discussion should focus on any differences noted.
  (N2.1, N2.2)

Journal

- Ask students to respond to the following:
  Insurance premiums are higher for young adult males than for young adult females. Do you agree or disagree with this practice? Justify your response.
  (N2.2)

Paper and Pencil

- Ask students to determine the cost of a car that they would like to own. They should find the cost of the newest model of this vehicle. Ask them to complete the following table, comparing the new and used models, and answer the questions that follow.

	New Car	3-Year-Old Used Car
Make and model		
Advertised price		
Advantages		
Disadvantages		
How would you feel about driving this car?		
What would your friends say if they saw you driving this car?		

(i) Using your data, which vehicle would you like to own? Why?
(ii) A "demo" car is also an option. What is a demo car? What are the advantages and disadvantages of owning one? (N2.2)

Resources/Notes

Authorized Resource

Math at Work 12

4.1 Owning a Vehicle

Student Book (SB): pp. 170-189

Teacher's Resource (TR): pp. 137-149

Web Links

The following websites contain car prices:

www.vmrcanada.com

www.canadianblackbook.com

Number

Outcomes

Students will be expected to

N2 Continued ...

Achievement Indicator:

N2.3 *Solve, with or without technology, problems that involve the purchase, lease or lease to purchase a vehicle.*

Elaborations—Strategies for Learning and Teaching

Students should solve problems that involve calculating the total cost of purchasing a vehicle after taxes and fees have been applied. Using technology, they determine the monthly payment based on a given interest rate. Students should be aware that the financing interest rate is a compound rate and cannot be calculated in the same manner as the tax rate. For this reason, technology should be utilized to calculate a monthly payment. They should also be exposed to examples where a down payment or discount is applied. They should be cautioned that these are applied after taxes.

Students should explore the payment schedules available when purchasing a vehicle (i.e., monthly or bi-weekly) and assess how this affects payments. Bi-weekly payments are made more frequently, but are less than half of a monthly payment.

Students should also solve problems that involve leasing a vehicle. This involves:

- calculating the total monthly payment after taxes are applied
- calculating the first monthly payment, which may include delivery or licensing fees.

It is important for students to be aware that leased vehicles have a mileage allowance – any kilometres over this allowance will be charged at a fixed rate. The cost of extra kilometres driven is calculated when the vehicle is returned.

Students should determine the total cost of a leased vehicle over the lease term, considering the monthly lease payments and the cost of financing the residual value. If choosing to purchase the vehicle at the end of the lease, they will be expected to calculate the monthly payment using a given residual value and interest rate. The total cost of purchasing a vehicle should be compared with the total cost of leasing a vehicle to decide on the most economical option to acquire a vehicle.

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies

Performance

- Discuss available options when purchasing a new car. Brainstorm a list of "must have" and "nice to have" options. Ask students to find a newspaper ad advertising a car they would like to own. They should then visit the dealer website and determine the total cost and monthly payment of the vehicle, including the options they have determined to be a "must have." Ask them to compare the advertised price with the quote they have generated on the website.

  (N2.3)

Journal

- Ask students to respond to the following:

  You are purchasing a new vehicle. You are deciding between taking a full loan for the purchase cost or spending the $3000 in your savings account as a down payment.

  The purchase price is $18 000 (taxes included) and the car loan will be for five years at an annual interest rate of 3.9%.

  Using a sample calculation in your response, what will be your decision? What financial and personal factors should be considered?

  (N2.3, N2.4)

Resources/Notes

Authorized Resource

Math at Work 12

4.1 Owning a Vehicle

SB: pp. 170-189

TR: pp. 137-149

Number

Outcomes

Students will be expected to

N2 Continued ...

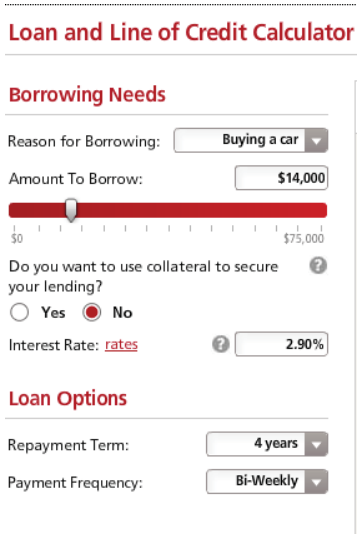
Achievement Indicators:

N2.3 *Continued*

N2.4 *Determine costs of operating a vehicle, including fixed costs, such as extended warranty, insurance and licensing, and ongoing costs, such as gas and maintenance.*

Elaborations—Strategies for Learning and Teaching

Students could use an online loan calculator to input purchase price, annual interest rate and the number of payments to calculate the monthly loan payment.



Alternatively, a graphing calculator could be used.

Students should calculate monthly and annual fixed costs of owning a vehicle when including fees, such as warranty, insurance and license fees. They could explore the relationship between the type of car and the fixed costs of owning that car.

Students should also consider the variable costs of owning a vehicle, including cost of fuel and maintenance. They will calculate annual maintenance costs, given a maintenance schedule with recommended repairs. Discuss the ongoing variable costs associated with maintaining a vehicle after the warranty has expired, particularly major repairs (e.g., transmission replacement). Students may consider whether it is more financially viable to continue maintaining an older vehicle without monthly payments or opting to purchase a new vehicle with warranty. Ideas generated through class discussion could be recorded in a chart such as shown below:

Old Vehicle		New Vehicle	
Advantages	Disadvantages	Advantages	Disadvantages

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to consider the following situation:

  You are purchasing a new vehicle, which you plan to drive 2000 km a month. You are given the option to purchase a "Freedom Driving" warranty package that costs $750 (plus tax), which will cover all maintenance for two years. The dealer provides you with the following maintenance schedule:

Maintenance	Schedule	Cost (plus tax)
Oil change	5000 km	$50
Tire rotation	5000 km	$25
Air filter	once yearly	$40

  Will you purchase the optional warranty package? Use the following calculations to justify your decision:

  (i) Calculate the cost of oil changes over a 2-year period.

  (ii) Calculate the cost of tire rotations over a 2-year period.

  (iii) Calculate the cost of replacing the air filter over a 2-year period. (N2.4)

- Ask students to consider the following:

  Francois is trying to decide whether to buy a hybrid car. He researches a hybrid car and records his findings:

Car	Cost	Fuel Economy (city)	Distance Driven/Year
Gas	$25 995	9.2 L/100 km	22 000 km
Hybrid	$30 995	5.1 L/100 km	22 000 km

  Complete the following table to determine which car would be the most economical option over a five-year period.

Calculation (5-year period)	Gas	Hybrid
# of km		
# of L of fuel		
cost of fuel at $1.35/L		

  (N2.4)

Presentation

- Ask students to determine whether a hybrid vehicle is worth the extra cost. They should find the base price and city fuel economy of a hybrid vehicle and compare them with a similar gas model (e.g., Toyota Camry vs. Camry Hybrid). Using the current cost of gasoline, have students determine the number of kilometres that would need to be driven to save the equivalent of the difference of the base model prices (excluding taxes). Students can present their findings to the class using a method of their choice. (N2.4)

Resources/Notes

Web Links

Many financial institutions have online loan calculators:

- *www4.bmo.com/popup/loans/Calculator.html*
- *cibc.com/ca/loans/calculators.html*
- *tdcanadatrust.com/loanpaymentcalc.form?lang=en*

Authorized Resource

Math at Work 12

4.2 Operating a Vehicle

SB: pp. 190-201

TR: pp. 150-160

Number

Outcomes

Students will be expected to

N3 Critique the viability of small business options by considering:

- **expenses**
- **sales**
- **profit or loss.**

[C, CN, R]

Elaborations—Strategies for Learning and Teaching

In today's society, it is important for students to develop an understanding of the business world and to develop critical thinking skills that will assist them in future entrepreneurial opportunities. Students should consider factors that affect profitability beyond revenue and expenses.

Concepts previously studied in this course, including recognizing patterns and trends, interpreting graphs and solving problems when provided with a table of values or graph (A1), will be built upon as students critique the viability of small business options.

Achievement Indicators:

N3.1 *Identify expenses in operating a small business.*

Students should identify expenses for a variety of small businesses. Fixed and variable expenses were introduced to students earlier in this chapter. Ask students to discuss start-up costs and on-going costs.

N3.2 *Identify feasible small business options for a given community.*

Students should consider a variety of factors when evaluating the feasibility of small business options in a given community. Possible factors to consider are:

- number of customers
- set-up or equipment costs
- value of goods or services
- weather
- number of employees required
- competition.

N3.3 *Generate options that might improve the profitability of a small business.*

Students should recognize that improved profitability could be achieved by increasing revenue or decreasing expenses. Increasing prices could theoretically generate more income, but they should also realize that this may exclude some customers. Extending operating hours and advertising may also generate more income, but this requires spending additional funds as well. In order to decrease expenses, students may suggest ways to minimize operating costs, such as finding a cheaper source for products or decreasing the number of employees. Advantages and disadvantages of each option should be discussed.

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to generate a table of expenses (fixed and variable) for the following businesses:
  (i) snow removal service
  (ii) summer daycare
  (iii) electronics repair

  (N3.1)

- Ask students to suggest ways that Lucy could decrease the following start-up costs associated with her flea market booth that she uses to sell quilts:
  (i) $750.00 for fabric
  (ii) $300.00 for a display rack
  (iii) $110.00 for a sign to display at the booth
  (iv) $30.00 to rent a cash register
  (v) $25.00 for booth rental fee

  (N3.3)

Performance

- Ask students to prepare a business proposal to an organization in their community that has offered to provide $5000 to a student or group with the most feasible business plan. Students may consider whether there is a demand for this good or service, any competing businesses in the area, the price of the good or service, and how long this business may be viable. Students could be interviewed to determine who will be awarded the funding.

  (N3.2)

Journal

- Ask students to respond to the following:

  Jill has a lawn care business during the months of July and August. Business has been booming to the point that she has been turning away clients. What recommendations would you make for Jill to improve profitability? Discuss the advantages and disadvantages of each recommendation.

  (N3.3)

Resources/Notes

Authorized Resource

Math at Work 12

4.3 Operating a Small Business

SB: pp. 202-215

TR: pp. 161-173

Note

Chapter Project: p. 220

This provides a good means of assessing outcome N3.

Number

Outcomes	Elaborations—Strategies for Learning and Teaching
Students will be expected to **N3 Continued ...**	
Achievement Indicators:	
N3.4 *Determine the profit, loss and break-even point for a small business.*	Students should calculate sales revenue based on the number of items sold or services provided at a given price. They should also calculate the expenses associated with providing each good or service. Questions such as the following could be used to calculate sales revenue based on items sold: • Sarah charges $65.00 for each print sold in her studio. What is Sarah's sales revenue if she sells 50 prints? • If it costs Sarah $17.95 to produce each print, what are the expenses required to produce 50 prints? Students calculate total expenses and revenue and use these values to determine the break-even point for a small business. • Assuming that Sarah makes 50 prints, how many does she need to sell in order to break even? Students are also required to determine net income (profit or loss) based on revenue and expenses presented in a table or graph. They should be able to draw a graph when provided with a table of values and determine the break-even point from the graph. Caution them against assuming that the month with the greatest revenue is always the month with the greatest profit.
N3.5 *Explain factors, such as seasonal variations and hours of operation, that might impact the profitability of a small business.*	Students should be exposed to questions regarding seasonal small businesses and asked to evaluate the profitability of the business over the full year. They should also explore how increasing or decreasing the hours of operation may impact the profitability of a small business.
N1 Analyze puzzles and games that involve logical reasoning, using problem-solving strategies. **[C, N, PS, R]**	As students explore small business options, this would be a good opportunity to revisit puzzles and games. *Tycoon Games* are a collection of games in which the player has to successfully run a business. Players must keep their workers happy, their clients coming back, and their resources properly functioning. They are available as online games and apps.

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to consider this situation and answer the questions that follow:

  Patrick started an online business selling graphic T-shirts. He charges $19.99 per shirt. It costs Patrick $10.50 to produce each T-shirt. His operating costs are $451 per month.

  (i) In one month, he made 75 T-shirts but he only sold 60 of them. Has Patrick covered his expenses? What is his net income?
  (ii) What must he have charged for his T-shirts in order to break even?
  (iii) If he had only produced 60 T-shirts, what would be Patrick's net income?

  (N3.4)

- Ask students to analyze the revenue and expenses for Donnie's small business and answer the questions that follow:

Month	Revenue	Expenses
January	$400	$85
February	$350	$60
March	$275	$55
April	$475	$90
May	$540	$110
June	$710	$140
July	$710	$125
August	$680	$110
September	$500	$95
October	$425	$85
November	$345	$70
December	$400	$75

  (i) Draw a line graph showing both revenue and expenses for Donnie's business over the year.
  (ii) Write a description of Donnie's business.
  (iii) Which month was the most profitable? Which month was the least profitable?
  (iv) What type of good or service might Donnie provide?
  (v) How does this affect the profitability over the year?
  (vi) Suggest factors that Donnie may consider to improve the profitability of his small business.

  (N3.5)

Resources/Notes

Authorized Resource

Math at Work 12

4.3 Operating a Small Business

SB: pp. 202-215

TR: pp. 161-173

Games and Puzzles

Mind Your Business

SB: p. 221

TR: pp. 179-180

BLM: 4-9

Properties of Figures

Suggested Time: 16 Hours

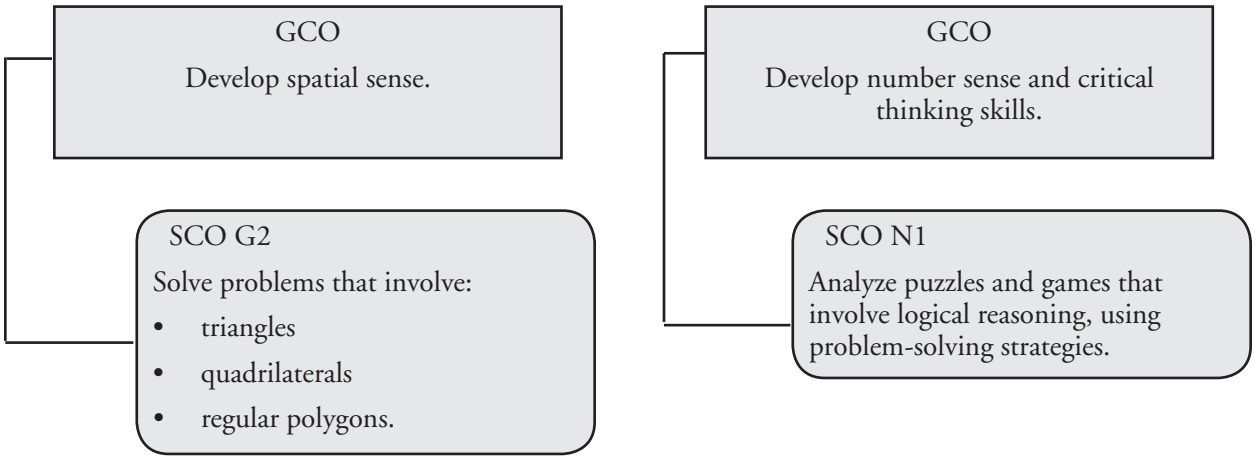
Unit Overview

Focus and Context

In this unit, students explore properties of geometric figures. They identify the properties of triangles, and classify them based on angle measures and side lengths. They distinguish between types of quadrilaterals based on side lengths, angle measures, diagonal lengths, and angles of intersection of diagonals.

Properties of regular polygons, including interior angle measures and sums, are explored. Students are exposed to real-world applications of triangles, quadrilaterals and regular polygons and consider uses of these geometric shapes.

Outcomes Framework



Mathematical Processes

[C] Communication
[CN] Connections
[ME] Mathematical Models and Estimation
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

SCO Continuum

Mathematics 1202	Mathematics 2202	Mathematics 3202
Geometry		
G5. Solve problems that involve parallel, perpendicular and transversal lines, and pairs of angles formed between them. [C, CN, PS, V] G6. Demonstrate an understanding of angles, including acute, right, obtuse, straight and reflex, by: • drawing • replicating and constructing • bisecting • solving problems. [C, ME, PS, T, V]	not addressed	G2. Solve problems that involve: • triangles • quadrilaterals • regular polygons. [C, CN, PS, V]
Number		
G1. Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies. [C, CN, PS, R]	N1. Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies. [C, CN, PS, R]	N1. Analyze puzzles and games that involve logical reasoning, using problem-solving strategies. [C, CN, PS, R]

Geometry

Outcomes

Students will be expected to

G2 Solve problems that involve:

- **triangles**
- **quadrilaterals**
- **regular polygons.**

[C, CN, PS, V]

Achievement Indicator:

G2.1 *Describe and illustrate angle properties of triangles.*

Elaborations—Strategies for Learning and Teaching

In this unit, students solve problems that involve the properties of polygons. They have not had any recent exposure to 2-D geometry. A review of definitions and characteristics of basic geometric shapes, as well as skills such as naming line segments and angles, will be necessary. In Grade 8, students explored the relationship between angles in polygons and their ability to tessellate (8SS6). In Grade 9, they worked with line symmetry (9SS5). Students worked with acute, right, obtuse, straight and reflex angles in Mathematics 1202 (G6). They will now further explore these topics and apply properties of triangles, quadrilaterals and regular polygons in contextual situations.

Students have had limited recent exposure to the terminology of geometry and, therefore, a review will be necessary before properties are explored. Students will have worked extensively with right triangles.

Students should investigate the classification of triangles based on side lengths:

- Equilateral – three equal sides
- Isosceles – 2 equal sides
- Scalene – no equal sides

For those students who struggle with remembering the names for each triangle, a strategy such as the following may help. Put the three triangle names in alphabetical order. Then apply the 3-2-1 rule: the first triangle (equilateral) has 3 equal sides, the second (isosceles) has 2 equal sides, and the third (scalene) has 1, or no equal sides.

Students should also identify whether the angles of a triangle are acute, right or obtuse.

The following properties of triangles should be explored:

- The sum of the angles in a triangle is 180°.
- An equilateral triangle has 3 equal angles which are acute.
- An isosceles triangle has two equal angles which are acute. The third angle could be acute, right or obtuse.
- A scalene triangle has no equal angles.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Portfolio

- Students could create a three tab foldable to classify the type of triangle and corresponding properties. On each of the outside panels, they write the name of the triangle: equilateral, isosceles and scalene. On the corresponding inside panel, students sketch the triangle and state the angle and side properties. They may also find it helpful to use colour to highlight congruencies (e.g., the same colour for the 3 sides and the same colour for the 3 angles of an equilateral triangle, the same colour for 2 sides and the same colour for 2 angles of an isosceles triangle, etc.)

(G2.1)

Performance

- Students could work together to create a word wall for the unit. Prepare strips of card stock or recipe cards for the word wall items. Students could first independently skim and scan sections of the text for unfamiliar words and symbols and record their words one per card. They should then work in small groups to compare cards and compile a master collection to be posted in the classroom.

  Alternatively, this could be used as a unit project where terms are added to the wall when introduced.

(G2)

- Students could sketch sample triangles on sticky notes and include segment congruency marks. In small groups, ask them to sort them by type and create a label for each set of triangles. The sorting could be done on a wall or desktop.

(G2.1)

Resources/Notes

Authorized Resource

Math at Work 12

5.1 Angle Properties of Polygons

Student Book (SB): pp. 226-237

Teacher's Resource (TR): pp. 189-197

Geometry

Outcomes

Students will be expected to

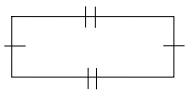
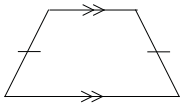
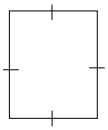
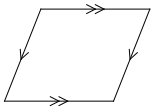
G2 Continued ...

Achievement Indicator:

G2.2 *Describe and illustrate angle properties of quadrilaterals.*

Elaborations—Strategies for Learning and Teaching

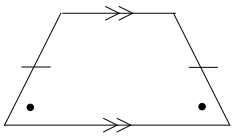
Before exploring angle properties of quadrilaterals, a review of different types of quadrilaterals is necessary. In this unit, students work with parallelograms, rectangles, squares, and isosceles trapezoids.

Rectangle	
Isosceles Trapezoid	
Square	
Parallelogram	

In Mathematics 1202, students worked with parallel lines with a focus on angle relationships that occur with parallel lines cut by a transversal (G5). Revisit the definition of parallel lines here. The following properties of quadrilaterals should be emphasized:

- The sum of the angles in a quadrilateral is 360°.
- The opposite angles of a parallelogram are equal.
- Squares and rectangles have four right angles.

Students should also recognize that the two angles opposite the equal sides of an isosceles trapezoid are equal.

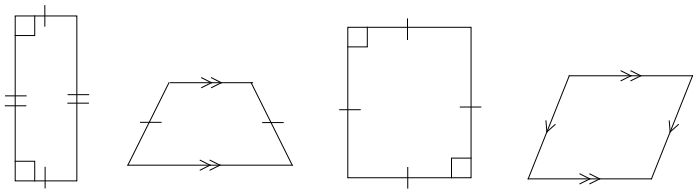


General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to classify each quadrilateral and to identify equal angles.



(G2.2)

Performance

- Ask students to create a four door foldable and label each outside flap with:
  - rectangle
  - square
  - isosceles trapezoid
  - parallelogram

  On the inside, students may sketch each figure and colour sets of equal angles (e.g., 4 angles of the same colour for a square and rectangle, pairs of the same coloured angles for an isosceles trapezoid and a parallelogram).

(G2.2)

Resources/Notes

Authorized Resource

Math at Work 12

5.1 Angle Properties of Polygons

SB: pp. 226-237

TR: pp. 189-197

Geometry

Outcomes

Students will be expected to

G2 Continued ...

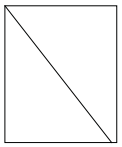
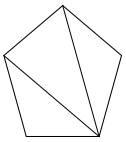
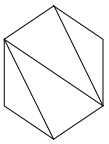
Achievement Indicator:

G2.3 *Describe and illustrate angle properties of regular polygons.*

Elaborations—Strategies for Learning and Teaching

Students described and compared the sides and angles of regular and irregular polygons in Grade 6. In this unit, they only work with regular polygons. A review of the definition and properties of regular polygons may be necessary. They should explore the sum of the interior angles of a regular polygon and determine the measure of each angle.

To discover the relationship between the sum of the interior angles and the number of sides in a regular polygon, students can separate each polygon into triangles by drawing diagonals. They should be aware that the sum of the angles in a triangle is 180°. A table such as the following could be used to help them with their investigation. They should note that, when dividing the polygon into triangles, each vertex of a triangle must be a vertex of the original polygon.

# of sides	Diagram	# of triangles formed	Sum of the angles
4		2	360°
5		3	540°
6		4	720°

The focus of this investigation is for students to recognize that the sum of the angles increases by 180° as the number of sides increases by one. They should also observe that the number of triangles formed is always two less than the number of sides in the polygon. The formula for the sum of the measures of the interior angles of a polygon could then be used:

$S = 180(n - 2)$, where S represents the sum of the interior angles and n is the number of sides of the polygon.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Performance

- Ask students to find an "eye contact partner" by standing up and making eye contact with someone who is not sitting near them. Once the partners move together, they should decide who will be Partner A and who will be Partner B. Give the class the following discussion prompt:

  Talk about what you know about the angle properties of triangles, quadrilaterals and regular polygons. What are the most important things you have learned? If there are areas you are still struggling with, talk them through for your partner to address when it is his or her turn.

  One partner talks for exactly one minute while the other partner listens. After one minute, partners switch roles and repeat. At the next switch, the first partner talks for 30 seconds, followed by another switch. At the end of the activity, volunteers could share with the class.

  (G2.1, G2.2, G2.3)

- In small groups and given a variety of regular polygons made of different colours of card stock, ask students to cut them into triangles using the vertices as end points. Once the number of triangles "contained" in each is discovered, ask students to determine the sum of the interior angles. Students may reassemble and mount the exploded polygon on paper above their indication of the angle measure.

  (G2.3)

Resources/Notes

Authorized Resource

Math at Work 12

5.1 Angle Properties of Polygons

SB: pp. 226-237

TR: pp. 189-197

Geometry

Outcomes

Students will be expected to

G2 Continued ...

Achievement Indicators:

G2.3 *Continued*

G2.4 *Explain, using examples, why a given property does or does not apply to certain polygons.*

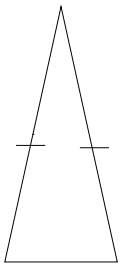
Elaborations—Strategies for Learning and Teaching

If the polygon is regular, the formula can also be used to determine the measure of each interior angle:
$M = \frac{180(n-2)}{n}$, where M is the measure of the angle and n is the number of sides of the polygon.

Another property of some regular polygons is the ability to tessellate. A tessellation is created when a shape is repeated over and over again covering a plane without any gaps or overlaps. Although this concept was introduced in Grade 8 (8SS6), a review of tessellations of basic polygons is necessary. Students should be given the opportunity to work with concrete materials to observe how the shapes either fit together, leave gaps open or overlap each other. The focus here is on determining if a given polygon will tessellate. Students are not expected to work with polygon combinations or to create tessellations. Pattern blocks may be useful for this exploration.

Students should recognize that the measure of the angle formula does not work for all polygons that have been explored. It cannot be applied for the irregular polygons explored (i.e., triangles, scalene triangles, parallelograms, isosceles trapezoids and rectangles).

The formula for the sum of the angles in a polygon will work for all polygons explored throughout this unit. Teachers could discuss with students that the measure of the angle formula implies that angles of the polygon are equal, which in turn implies the side lengths are equal. Consequently, this formula only works with regular polygons. Applying the formula to an isosceles triangle, for example, results in each angle measuring 60°. This makes the triangle equilateral rather than isosceles.



$$M = \frac{180(3-2)}{3}$$
$$M = \frac{180}{3}$$
$$M = 60°$$

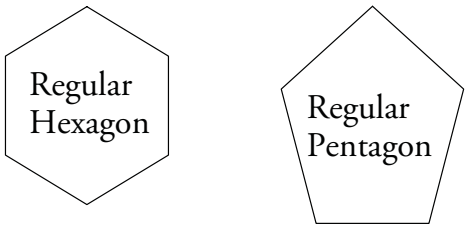
Students should also come to realize that all triangles, squares and regular hexagons will tessellate. This is because at any point where vertices meet, the sum of the angles is 360°.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to determine if these shapes tessellate.



Students can create the shape on an index card, cut it out, and then trace, slide and repeat to see if it tessellates.

(G2.3)

- Ask students to determine the sum of the interior angles and the measure of each interior angle of the following polygons.
  (i) 7-sided polygon
  (ii) 10-sided polygon
  (iii) 12-sided polygon

(G2.3)

Interview

- Ask students to explain why $M = \frac{180(n-2)}{n}$ cannot be used to determine the measure of the interior angle of a parallelogram.

(G2.4)

Resources/Notes

Authorized Resource

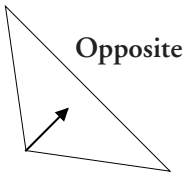
Math at Work 12

5.1 Angle Properties of Polygons

SB: pp. 226-237

TR: pp. 189-197

Geometry

Outcomes	Elaborations—Strategies for Learning and Teaching
Students will be expected to **G2 Continued ...**	
Achievement Indicators:	
G2.5 *Describe and illustrate side length properties of triangles.*	Students should further explore the relationship between angle measures and side lengths in triangles. • The side opposite the largest angle is the longest side. • The side opposite the smallest angle is the smallest side. • The sum of any two sides must be greater than the length of the third side. It is important that students recognize the side opposite the angle being referenced. 
G2.6 *Describe and illustrate properties of quadrilaterals in terms of side lengths, diagonal lengths and angles of intersection.*	Additionally, students should explore the following properties of quadrilaterals: • Both pairs of opposite sides of a rectangle are parallel. • Both pairs of opposite sides of a parallelogram are equal. • The diagonals of a rectangle are equal. • The diagonals of a square are equal. • The diagonals of an isosceles trapezoid are equal. • The diagonals of a parallelogram are not equal. The longest diagonal will be opposite the largest angle. • The diagonals of all quadrilaterals studied will intersect at the midpoint of the diagonals. • The diagonals of a square will intersect to form right angles. • The sum of the angles where the diagonals of a quadrilateral intersect will be 360°.
G2.7 *Describe and illustrate the diagonal properties of regular polygons.*	Students should also explore the number of diagonals in a regular polygon by drawing the diagonals from each vertex and counting the total number. It is not necessary to introduce students to the formula for determining the number of diagonals in a regular polygon.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Interview

- Present students with a variety of triangles with the angle measures included and ask them to identify the largest side or the smallest side. They should explain how they made their choice.

(G2.5)

- Present students with triangles which are labelled with side lengths. Ask them to explain if the triangles are possible.

(G2.5)

- Present students with two regular polygons and ask them to explain which polygon will have the greatest number of diagonals.

(G2.7)

Journal

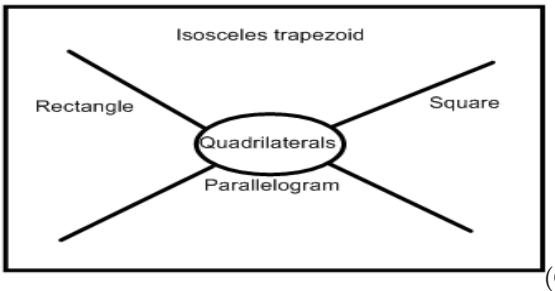
- Ask students to respond to the following:

  Sketch three triangles that will have a perimeter of 24 cm. Explain why these triangles are possible using triangle properties.

(G2.5)

Portfolio

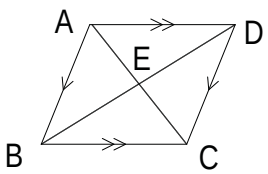
- Ask students to complete a placemat graphic organizer on the properties of quadrilaterals.



(G2.2,G2.6)

Paper and Pencil

- Ask students to use the properties of quadrilaterals to determine the missing measurements.



AD=8cm
BC= ?
AE=2cm
EC= ?

(G2.2, G2.6)

Resources/Notes

Authorized Resource

Math at Work 12

5.2 Side Lengths and Diagonal Properties of Polygons

SB: pp. 238-247

TR: pp. 198-206

Geometry

Outcomes

Students will be expected to

G2 Continued ...

Achievement Indicator:

G2.8 *Describe and illustrate line symmetry in triangles, quadrilaterals and regular polygons.*

Elaborations—Strategies for Learning and Teaching

Students will now explore line symmetry in triangles, quadrilaterals and regular polygons. A 2-D shape has line symmetry if one half of the shape is a reflection of the other half. The reflection occurs across a line. The line of symmetry, or line of reflection, can be horizontal, vertical or oblique, and may or may not be part of the diagram itself. To help explain line symmetry, students should view examples and non-examples. Another approach is to have students fold a sheet of paper in half and cut out a shape of their choosing. When they open the paper, the fold line will be a line of symmetry. Alternatively, transparent mirrors could be used. If the shape is symmetrical where the mirror has been placed, the image of one side of the shape will fall right on top of the other side of the shape.

Students should be given an opportunity to investigate the number of lines of symmetry that exist in various 2-D shapes, including triangles, quadrilaterals and regular polygons. They should also explore symmetry in letters, pictures, logos, etc. In examining shapes to be classified based on the number of lines of symmetry, it is important to include shapes that are asymmetrical. Students can complete shapes and designs with line symmetry using square tiles or pattern blocks, folded paper, transparent mirrors, grid paper, or technology tools such as a drawing program or dynamic geometry software.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Performance

- In groups of two, one student uses rubber bands to create half of a polygon on a geoboard. Using another rubber band, the second student will complete the other half of the polygon. Students should discuss if the polygon is regular and if there is another type of polygon that can be formed.

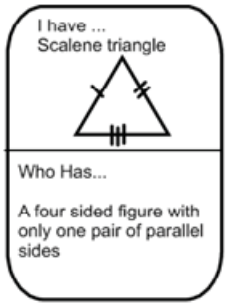
  (G2.8)

- Students could tile a section of the classroom floor using models of coloured square floor tiles. A section of the classroom is taped off. Students make a design where floor tiles of the same colour cannot touch. The tiles can also be cut into triangles.

  Students could choose to create a pattern using the six colours. The pattern could be repeated to cover the designated area.

  (G2.8)

- Students could play "I Have...Who Has" for properties of triangles, quadrilaterals and regular polygons. Provide them with a loop card as shown below. Choose a student to start and read the "Who Has..." part of the card aloud. A student will respond "I have..." answering with the correct polygon described by the properties. The student continues to read "Who Has..." This will continue until all of the class has read their cards.



  This could be scaffolded by having pairs of students holding a card or cards for the activity.

  (G2.1 to G2.8)

Resources/Notes

Authorized Resource

Math at Work 12

5.3 Symmetry

SB: pp. 248-257

TR: pp. 207-214

Chapter Project

Logo Pro

SB: p. 262

TR: pp. 217-218

Web Links

- *www.brendenisteaching.com/gen/myloops/*
- *www.senteacher.org/Worksheet/46/Loopcards.xhtml*

Templates for creating loop cards can be found at these sites.

Geometry

Outcomes

Students will be expected to

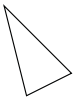
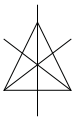
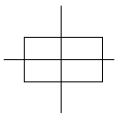
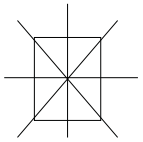
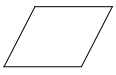
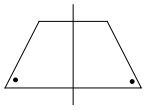
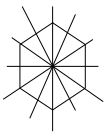
G2 Continued ...

Achievement Indicators:

G2.8 *Continued*

Elaborations—Strategies for Learning and Teaching

Relating a line of symmetry to a line of reflection should enable students to complete a figure, describe the completed shape, and describe the reflection.

Figure Name	Diagram	# of lines of symmetry
Scalene Triangle		0
Isosceles Triangle		1
Equilateral Triangle		3
Rectangle		2
Square		4
Parallelogram		0
Isosceles Trapezoid		1
Hexagon		6

Students should note that the number of lines of symmetry in a regular polygon (which includes the equilateral triangle and the square) is equal to the number of vertices.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Journal

- Ask students to explain if a rectangle and a parallelogram have the same number of lines of symmetry.

(G2.8)

- Ask students to discuss why a rectangle has two lines of symmetry, while a square has four lines of symmetry.

(G2.8)

Paper and Pencil

- Ask students to determine the number of lines of symmetry in each of the following regular polygons.

  (i) 7-sided polygon
  (ii) 10-sided polygon
  (iii) 12-sided polygon

(G2.8)

Performance

- Students could fold paper cutout versions of polygons to discover lines of symmetry. They should trace each successful fold with a different colour to determine the number of lines of symmetry.

(G2.8)

Resources/Notes

Authorized Resource

Math at Work 12

5.3 Symmetry

SB: pp. 248-257

TR: pp. 207-214

Geometry

Outcomes

Students will be expected to

G2 Continued ...

Achievement Indicators:

G2.9 *Identify and explain an application of the properties of polygons in construction, industrial, commercial, domestic and artistic contexts.*

G2.10 *Solve a contextual problem that involves the application of the properties of polygons.*

Elaborations—Strategies for Learning and Teaching

There is opportunity for real-life applications using the properties of polygons. Students should explore flooring tiles, cutting construction materials, quilting, squaring frames on buildings, and building design. They should explore different types of art such as paintings, jewellery, quilts, tiles, murals, and cultural artwork. Students could analyze pictures, logos, flags, signs, playing cards, and kaleidoscopes. Computer assisted software could be useful when experimenting with properties of polygons in designs or photos.

In all cases, an effort should be made to contextualize problems using domestic, trades and occupational examples. Students should answer questions such as:

- How do you know the frame is "square"?
- How can you locate the center of a room?
- Will the plans for a roof truss for a house work?

Students could go on a "math walk" throughout the building or neighbourhood to take photographs of polygons. They should write a description of the polygons in each photo, including their properties and their purpose. Ask them to consider if the polygons are functional, visually appealing, or both.

N1 Analyze puzzles and games that involve logical reasoning, using problem-solving strategies.

[C, CN, PS, R]

This would be a good time to incorporate puzzles and games involving logical reasoning. Refer back to pp. 38-47 for additional information.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

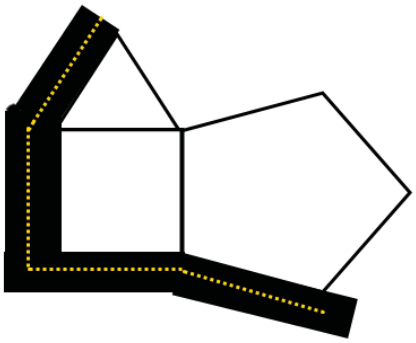
Paper and Pencil

- Ask students to answer the following:

  (i) A local contractor was hired to build a shed 6 m wide by 8 m long. The four walls of the shed did not look even (plumb) to the homeowner. How could the homeowner check that the walls were "square"?

  (G2.9, G2.10)

  (ii) A surveyor is laying out a new subdivision, divided into four lots. Three of the lots are shown in the diagram.



  (a) If a fourth lot is added that borders the pentagon and the triangle, what will be the angle of the new lot at the corner shared by the three lots?

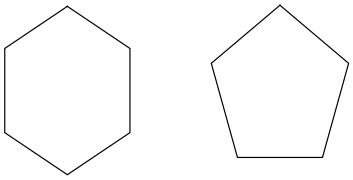
  (b) What are possible shapes of the fourth lot?

  (G2.9, G2.10)

Journal

- Ask students to respond to the following:

  Katie's mom has to choose between the following two ceramic tiles to cover the bathroom floor.



  Explain to her mom which tile sample she should use and why.

  (G2.9, G2.10)

Resources/Notes

Authorized Resource

Math at Work 12

5.1 Angle Properties of Polygons

SB: pp. 226-237

TR: pp. 189-197

5.2 Side Lengths and Diagonal Properties of Polygons

SB: pp. 238-247

TR: pp. 198-206

5.3 Symmetry

SB: pp. 248-257

TR: pp. 207-214

Games and Puzzles

Shape Search

SB: p. 263

TR: pp. 219

BLM 5-12

Transformations

Suggested Time: 15 Hours

Unit Overview

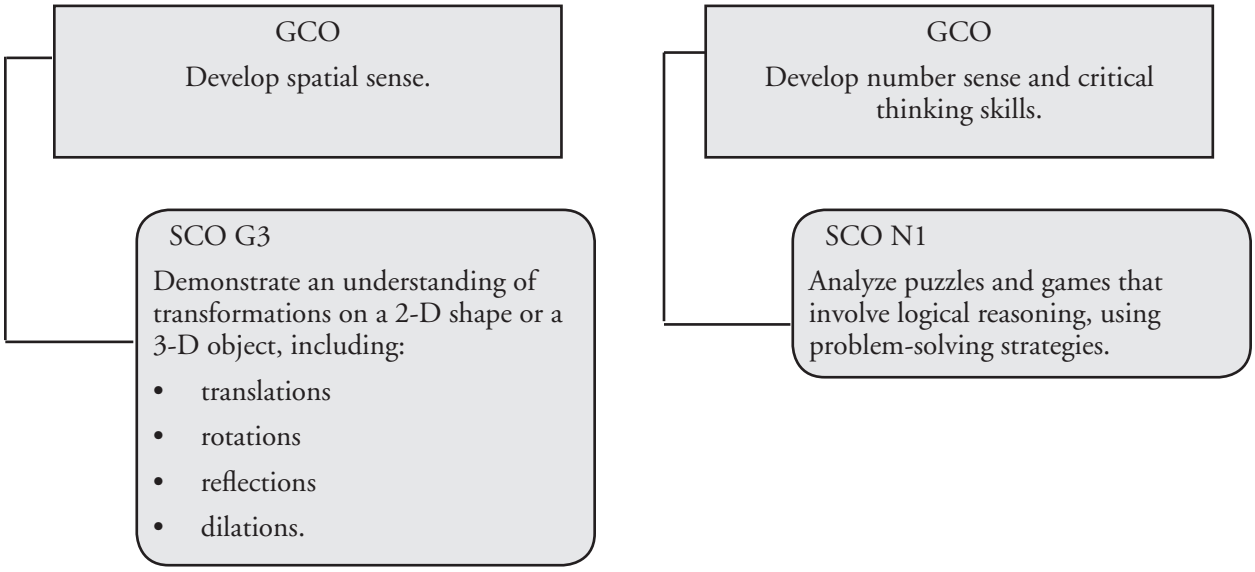
Focus and Context

Understanding transformations allows students to apply the concepts of similarity, proportion and angles in practical settings. Working with dilations, translations, reflections, and rotations will enable students to describe the transformations that occur throughout their everyday lives.

In this unit, students will identify and draw transformations performed on two-dimensional shapes and three-dimensional objects. They will also draw and analyze 2-D shapes that result from a combination of successive transformations. Designs in all four quadrants of a coordinate plane will be created, analyzed and described.

This topic gives students the opportunity to improve their spatial sense. Many jobs, from stocking shelves effectively to designing a room, require spatial reasoning.

Outcomes Framework



Mathematical Processes

[C] Communication
[CN] Connections
[ME] Mathematical Modeling and Estimation
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

SCO Continuum

Mathematics 1202	Mathematics 2202	Mathematics 3202
Geometry		
G3. Demonstrate an understanding of similarity of convex polygons, including regular and irregular polygons. [C, CN, PS, V]	G2. Solve problems that involve scale. [PS, R, T, V]	G3. Demonstrate an understanding of transformations on a 2-D shape or a 3-D object, including: • translations • rotations • reflections • dilations. [C, CN, R, T, V]
Number		
G1. Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies. [C, CN, PS, R]	N1. Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies. [C, CN, PS, R]	N1. Analyze puzzles and games that involve logical reasoning, using problem-solving strategies. [C, CN, PS, R]

Geometry

Outcomes

Students will be expected to

G3 Demonstrate an understanding of transformations on a 2-D shape or a 3-D object, including:

- **translations**
- **rotations**
- **reflections**
- **dilations.**

[C, CN, R, T, V]

Achievement Indicators:

G3.1 *Determine and explain whether a given image is a dilation of another given shape, using the concept of similarity.*

G3.2 *Draw, with or without technology, a dilation image for a given 2-D shape or 3-D object, and explain how the original 2-D shape or 3-D object and its image are proportional.*

Elaborations—Strategies for Learning and Teaching

In this unit, students will explore transformations of two-dimensional shapes and three-dimensionsal objects. They will discover that a transformation is a process whereby a set of points change. The change can involve location, size, or both. They will first work with transformations independently and then progress to a combination of transformations in all four quadrants of the Cartesian plane as well as other mediums.

Students should be introduced to the notation used when applying transformations. The image of point A, for example, after a transformation of any type is labelled point A′.

In Mathematics 1202, students were introduced to the concept of similarity (G2). In Grade 9, they worked with reductions and enlargements of 2-D shapes (9SS4), as well as properties of similar polygons (9SS3).

The criteria required for similarity in shapes (i.e., corresponding angles are congruent and corresponding sides are proportional) should be reviewed. Introduce students to the concept of dilation as a transformation in which an object is enlarged or reduced by a constant factor. This will be their first exposure to the term dilation.

In Mathematics 2202, students were introduced to the concept of scale (G2). Providing them with an opportunity to explore real world examples of scale diagrams, they should discuss the concept of a scale factor, be able to determine the scale factor, and use it to create enlargements and reductions. Students will also draw dilations of 2-D shapes and 3-D objects, with or without technology. They could use isometric dot paper when drawing their dilations of objects.

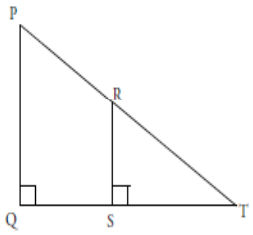
Students should also be aware of the effect of the magnitude of a scale factor. They should be asking themselves questions such as: What happens when a scale factor is greater than 1? Less than 1? A common student error occurs when they interchange the numerator and denominator while calculating scale factor. Understanding that for an enlargement the scale factor is greater than 1, and for a reduction the scale factor is less than 1, should help students avoid making that mistake.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to answer the questions that follow based on the diagram given.



(i) Which triangles are similar?

(ii) Measure the sides and determine the ratios of

(a) $\frac{PQ}{QT}, \frac{RS}{ST}$

(b) $\frac{PQ}{PT}, \frac{RS}{RT}$

(c) $\frac{QT}{PT}, \frac{ST}{RT}$

What do you notice about the values?

(iii) Is ΔTPQ a dilation of ΔTRS? Explain your reasoning.

(G3.1)

Resources/Notes

Authorized Resource

Math at Work 12

6.1 Dilations

Student Book (SB): pp. 268-281

Teacher's Resource (TR): pp. 230-236

Web Link

www.mathopenref.com/dilate.html

This site demonstrates the concepts of scale (proportionality), similarity and shape of a dilated image.

Geometry

Outcomes	Elaborations—Strategies for Learning and Teaching
Students will be expected to **G3 Continued ...**	
Achievement Indicators:	
G3.2 *Continued*	Discuss with students that like units are necessary when finding a scale factor. They could be asked to find the actual size, using the scale, or to convert the scale provided in one form to a different form. They could be asked a question such as the following: *If the scale is given as the ratio 1:50, how many metres does 7.5 cm represent?* To solve this problem, students might recognize that they could multiply 50 by 7.5 cm or they may set up the proportion: $\frac{1}{50} = \frac{7.5}{x}$ They should initially determine that x = 375. Since 7.5 is in cm, then 375 is also in cm, and when converted equals 3.75 m.
G3.3 *Identify and describe the applications of transformations in construction, industrial, commercial, domestic and artistic contexts.*	Students should be given an opportunity to explore dilations in the context of everyday experiences. Consider the following examples: • Construction: signs, high rise buildings, wood working, tiles • Industrial: blue prints, brick laying • Commercial: jewellery, logos • Domestic: quilts, playing cards • Artistic: paintings, murals, kaleidoscopes, synchronized sports
G3.4 *Solve a contextual problem that involves transformations.*	Students could use computer assisted software, such as Google Sketch-Up, Microsoft Publisher or Paint, or cell phone tools or apps to experiment with transformations of designs or photos. Ensure that they can recognize situations and use dilations to solve problems in everyday life. • Students could find a picture they would like to enlarge or reduce. They can then dilate photos by either using grid paper or using technology. • Each student could be given a section of a photo, sketch, map, etc. to enlarge to create a class mural.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Paper and Pencil

- Provide students with a series of shapes on a Cartesian plane and ask them to perform dilations, using scale factors greater than one and less than one. They should explain how the shapes and images are proportional.

(G3.2)

Observation

- Students could visit the following interactive website relating to transformations: *http://www.mathopenref.com/dilate.html*

  Ask the following questions to promote discussion:

  (i) What happens to the image when the scale factor is 2, 1, 0.5?
  (ii) What will happen to the image when the shape is changed?
  (iii) Predict what will happen to the image if you change one point to create a triangle.
  (iv) If the distance between O and B is 10, what is the distance between O and B′?

(G3.2)

Journal

- Ask students to respond to the following:

  Why do you think it is necessary to build a smaller model of a boat before building the actual boat?

(G3.3)

- Ask students to observe and record examples of dilations at school and at home.

(G3.3)

Performance

- Ask students to design a logo that includes geometric shapes. They should consider the following points to help them with their design:

  (i) Decide on the dimensions of an enlargement of the logo which would fit on a banner or billboard.
  (ii) Determine the scale factor.
  (iii) Create a business card using the logo by repeating the process for a reduction.

(G3.4)

Resources/Notes

Authorized Resource

Math at Work 12

6.1 Dilations

SB: pp. 268-281

TR: pp. 230-236

Geometry

Outcomes

Students will be expected to

G3 Continued ...

Achievement Indicators:

G3.5 *Draw the image of a 2-D shape that results from a given single transformation.*

G3.6 *Identify a single transformation that was performed, given the original 2-D shape or 3-D object and its image.*

G3.7 *Draw the image of a 2-D shape that results from a given combination of successive transformations.*

G3.3, G3.4 *Continued*

Elaborations—Strategies for Learning and Teaching

Unlike dilations, transformations such as translations, reflections and rotations do not change the size of the figure. Introduce students to the concept of a translation as a transformation that slides an image in a straight line without changing its size or orientation. Students were previously introduced to the concepts of plotting points on a Cartesian plane in Grade 7 (7SS4).

Most translations that students will be working with will be on a Cartesian plane but should not be limited to this medium. For example, quilts, tiles and stone pathways are also possible mediums.

Students should be able to:

- perform a horizontal, vertical or successive translation(s), given an image
- draw an image given the coordinates and perform a horizontal, vertical or successive translation(s).
- identify the translation(s) that have occurred given a 2-D shape or a 3-D object and their images.

The initial focus should be on shapes such as rectangles and triangles before more complex shapes are introduced.

Students should recognize situations whereby translations are used to solve problems in everyday life. For example:

- rearranging furniture in a given room
- playing games such as Battleship™, Checkers™, Chess, Tetris™, etc.
- creating ceramic tile patterns
- creating 3-D drawing

Ask students to create a design using translations and to then describe how the translations were used in their design.

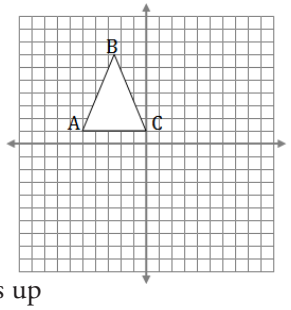
Students should work through a variety of examples where they follow a set of instructions involving successive transformations (e.g., a translation followed by a translation).

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Paper and Pencil

- Students should use the diagram below to complete the translations indicated:



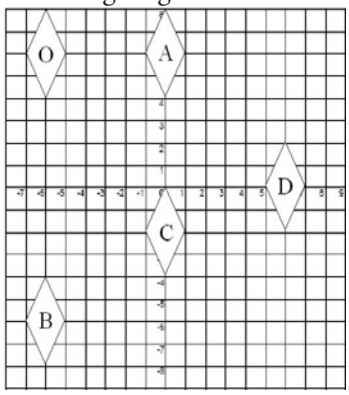
Translate ΔABC ….

(i) 3 units left and 2 units up

(ii) 4 units right and 1 unit down (G3.5, G3.7)

Interview

- Ask students to analyze the following diagram to answer the questions that follow:



(i) Identify the ordered pairs of the vertices for the object O and its images, A, B, C and D.

(ii) Describe the transformations required to move from O to each of the images. (G3.6, G3.7)

Performance

- Create a grid on the floor with masking tape. Use rope or coloured tape to place the axes. One student chooses a spot. Another student directs him/her to translate the position. This activity could progress to more than 3 students on the grid holding elastic to form a polygon. A different student directs them (as vertices) to "walk through" various transformations. (G3.7)
- Students could create an aisle and row plan for the classroom. Place all student names in a bag. Have one student select a name. They must describe the translation required for him or her to move to the selected student's seat. The displaced student then selects a name and describes the translation required to move to the selected student's seat. The cycle continues until all students have moved.

(G3.4, G3.7)

Resources/Notes

Authorized Resource

Math at Work 12

6.2 Translations

SB: pp. 282-291

TR: pp. 237-243

Web Link

http://mathopenref.com/translate.html

Using this interactive website, students can explore translations of polygons.

Geometry

Outcomes

Students will be expected to

G3 Continued ...

Achievement Indicators:

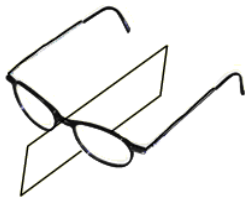
G3.8 *Explain the relationship between reflections and lines or planes of symmetry.*

G3.3, G3.4, G3.5, G3.6, G3.7 *Continued*

G3.9 *Create, analyze and describe designs, using translations, rotations and reflections in all four quadrants of a coordinate grid.*

Elaborations—Strategies for Learning and Teaching

In Grade 9, students were introduced to lines of symmetry (9SS5). Discuss with them that a reflection is a transformation in which the object is shown as its mirror image over a line of reflection. They should realize that corresponding points on both sides of that line are the same distance away from the line. The line of symmetry, or line of reflection, can be horizontal, vertical or oblique, and may or may not be part of the diagram itself. Students should be able to make the distinction that 2-D shapes have an axis of symmetry, while 3-D objects have a plane of symmetry. The figure below illustrates a 3-D plane of symmetry:



www.reciprocalnet.org/edumodules/symmetry/operations/reflection.html

When working with reflections, students should complete the following:

- Given a line of reflection, produce a reflection of an image.
- Given a line of reflection, produce a reflection image given the coordinates of an object.
- Given a 2-D shape or a 3-D object, identify the reflection(s),that have occurred.
- Given a 2-D shape or a 3-D object, identify the transformations that have occurred.

Provide students with an opportunity to recognize where reflections are used in everyday life. Ask them:

- why the reflection of the word AMBULANCE is written on the front of an ambulance.
- to identify and describe lines of reflection in different types of materials such as signs, tile patterns, patio stones, jewellery, crystals, quilts, playing cards, paintings, murals, kaleidoscopes, etc.
- to discuss reflections involved in creating symmetrical patterns in clothing or wood working projects.

Computer assisted software can be useful when experimenting with symmetry of designs or photos. This provides a good opportunity to introduce a cross-curricular project or activity.

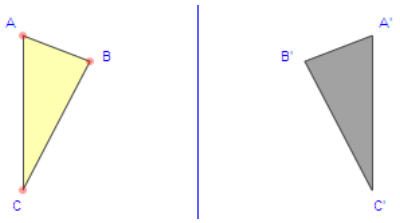
Students will work first with various types of transformations independently, and then with successive transformations (e.g., a reflection followed by a reflection), followed by a combination of transformations (e.g., a reflection followed by a translation).

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Paper and Pencil

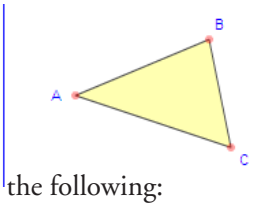
- Ask students to use the diagram to answer the questions that follow:



(i) Identify the type of transformation.
(ii) What is the relationship between ΔABC and $\Delta A'B'C'$?

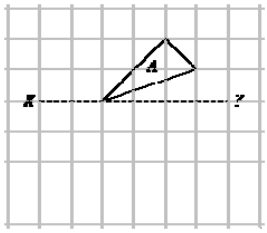
(G3.6, G3.8)

- Ask students to construct a reflection of the following image:



(G3.5)

- Ask students to answer the following:



(i) Reflect figure A in the line XY. Label the image A′.
(ii) Translate A′ two units right. (G3.5, G3.9)

Performance

- Ask students to use a digital camera or cell phone to take photographs of their faces. Instruct them to look directly at the camera and avoid tilting their heads. Using a drawing program, such as Paint Shop Pro or Adobe Photoshop, or cell phone tools or apps, students can then follow the steps outlined below:

(i) Crop the right side of the face.
(ii) Copy the remaining left side.
(iii) Paste a mirror image of the left side in the right-hand position.
(iv) Repeat the procedure to produce a mirror image of the right side of the face.
(v) Print the three photos.
(vi) Compare the "symmetrical" photos with the original. (G3.8)

- Ask students to find four flags that demonstrate line symmetry. (G3.9)

Resources/Notes

Authorized Resource

Math at Work 12

6.3 Reflections

SB: pp. 292-301

TR: pp. 244-250

Chapter Project

Make an Animation

SB: p. 320

TR: pp. 261-262

Geometry

Outcomes

Students will be expected to

G3 Continued ...

Achievement Indicators:

G3.3, G3.4, G3.5, G3.6, G3.7, G3.9 *Continued*

Elaborations—Strategies for Learning and Teaching

In Grade 9, students were introduced to line and rotation symmetry (9SS5). The focus here is on rotations. Discuss with students that a rotation is a transformation that moves an object around a fixed point, called the center of rotation. The majority of the rotations that students will be working with are on a Cartesian plane but examples should not be limited to this medium. Students could, for example, use trace paper, computer software or grid paper.

Students should initially work with rotations of 90°, 180° and 270° both clockwise and counterclockwise. They should explore various methods for completing rotations such as rotational rule or compass and protractor. When working with rotations, students should complete the following tasks:

- perform a specified rotation or a combination of successive transformations given any 2-D shape.
- draw an image given the coordinates and perform a specified rotation or a combination of successive transformations.
- identify the transformation(s) that have occurred given a 2-D shape or a 3-D object and their images.

Similar to previous transformations, students will work through examples that involve successive transformations. It is important for them to identify the order in which these transformations occur. Discuss with them that a different order of the same transformations may lead to a different result.

Many designs use rotational properties. Examples include:

- Ferris wheels, fans
- Ceramic tile and quilting patterns
- creating a 3-D drawing
- logos: recycling logo, Yin Yang logo

Students could look at a design and identify the center of rotation as well as the rotational angle.

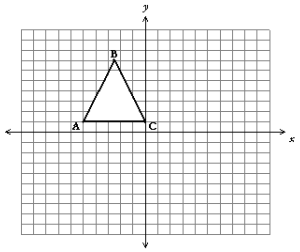
Ask students to create a design using rotations. They should describe how rotations are used to create the design using all four quadrants. Software applications such as Photoshop, Microsoft Publisher, Auto Cad, Sketch-Up, and cell phone tools have built in rotation capabilities. Students could use a digital image to explore the rotational tools of the software. They should then investigate the role of transformations in the creation of designs. Students could also create a design using a combination of successive transformations.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Paper and Pencil

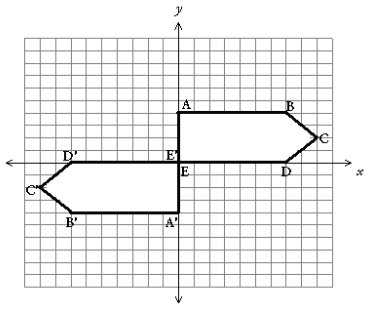
- Ask students to transform the shape as instructed using a method of their choice.



(i) Rotate 180° about (0, 0)
(ii) Rotate 90° clockwise about (0, 1)
(iii) Rotate 45° counter clockwise about the origin
(iv) Rotate 270° counter clockwise about (1, 3) and translate three units down.

(G3.5, G3.7)

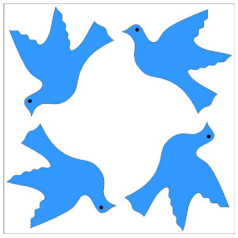
- Ask students to analyze the following diagram and identify the transformation.



(G3.6)

Observation

- Ask students to identify the center of rotation and the rotational angle in a design such as the following. They could place a spinner at the center of rotation to check the accuracy of their location.



http://www.kathykwylie.com/blog/wp-content/uploads/2011/03/Blue-Bird-Rotational-Symmetry.jpg

(G3.4, G3.6, G3.7)

- Students could look at selected work by M. C. Escher and identify the transformations used in his artwork. (G3.5, G3.6, G3.8)

Resources/Notes

Authorized Resource

Math at Work 12

6.4 Rotations

SB: pp. 302-315

TR: pp. 251-257

Web Link

www.mcescher.com/Gallery/gallery-symmetry.htm

Number

Outcomes

Students will be expected to

N1 Analyze puzzles and games that involve logical reasoning, using problem-solving strategies. [C, CN, PS, R]

Elaborations—Strategies for Learning and Teaching

Move It! ™ is a block-sliding puzzle, available as an app for mobile devices.

The objective is to move the red square to the exit. Blocks can be moved up, down, left and right. There must be open spaces in the direction the player is attempting to move.

As students play this game, ask them how it relates to the transformations they have studied in this unit. They should try to move to the exit in as few translations as possible.

Other similar puzzles include *Unblock Me*, *Sliding Block Puzzle*, *Shift It*, *Slide It*, and *Cohesian*.

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies

Performance

- Students draw two copies of a 3 × 3 grid on a piece of paper. On one of the copies, they write the numbers 1 to 8, leaving one of the squares blank. Ask them to cut out the numbers so that each one is on a separate piece of paper. They should then arrange them on top of the other grid, in the following pattern:

2		3
1	7	4
8	6	5

Students should move the numbers around by translating them up, down, left, or right into the empty square, until they have rearranged them into the following pattern:

1	2	3
4	5	6
7	8	

Ask them to count the number of translations required to rearrange the numbers. They should try to use the fewest number of moves possible.

Students could also create their own variation of this puzzle and then switch puzzles to solve another student's puzzle.

Resources/Notes

Authorized Resource

Math at Work 12

Games and Puzzles

Transformation Golf

SB: p. 321

TR: p. 263

BLM 6-10

Trigonometry

Suggested Time: 10 Hours

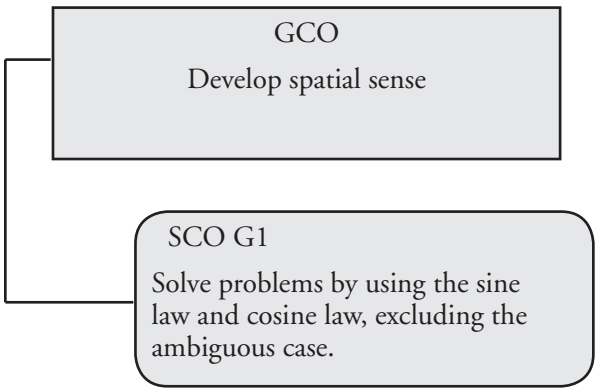
Unit Overview

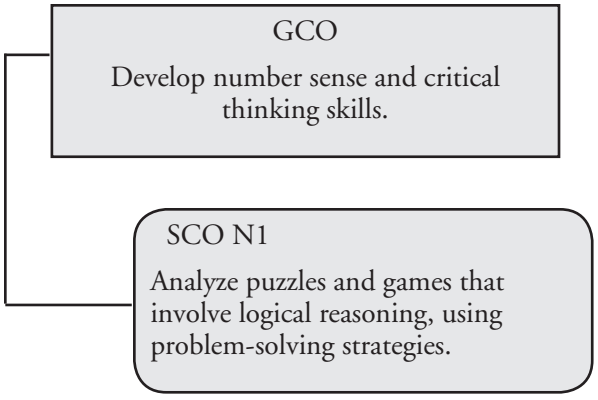
Focus and Context

In this unit, previous work with trigonometry is extended to include trigonometry of non-right triangles. Students explore the sine law and the cosine law, excluding the ambiguous case. They apply the formulas to determine unknown side lengths and angle measures.

Contexts such as construction, surveying, and navigation rely on trigonometry. While in many cases a triangle can be divided into right triangles, it can be much more efficient to work through a problem using oblique triangles and the sine and cosine laws.

Outcomes Framework





Mathematical Processes

[C] Communication
[CN] Connections
[ME] Mathematical Modeling and Estimation
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

SCO Continuum

Mathematics 1202	Mathematics 2202	Mathematics 3202
Geometry		
G4. Demonstrate an understanding of primary trigonometric ratios (sine, cosine, tangent) by: • applying similarity to right triangles • generalizing patterns from similar right triangles • applying the primary trigonometric ratios • solving problems. [CN, PS, R, T, V]	G1. Solve problems that involve two and three right triangles. [CN, PS, T, V]	G1. Solve problems by using the sine law and cosine law, excluding the ambiguous case. [CN, PS, V]
Number		
G1. Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies. [C, CN, PS, R]	N1. Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies. [C, CN, PS, R]	N1. Analyze puzzles and games that involve logical reasoning, using problem-solving strategies. [C, CN, PS, R]

Geometry

Outcomes

Students will be expected to

G1 Solve problems by using the sine law and cosine law, excluding the ambiguous case.

[CN, PS, V]

Achievement Indicator:

G1.1 *Solve a problem uisng the sine law, when a diagram is given.*

Elaborations—Strategies for Learning and Teaching

In Mathematics 1202, students used the three primary trigonometric ratios (sine, cosine and tangent) to determine the side lengths and angle measures in right triangles (G4). In Mathematics 2202, this was extended to include problems involving two and three right triangles (G1). In this unit, students will apply the Law of Sines and the Law of Cosines in various problem-solving situations.

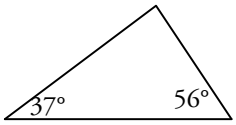
Students have used right triangle trigonometry to solve problems involving right triangles. A review of correctly labelling sides in relation to opposite angles may be needed. They will now solve oblique triangles, using the sine law to determine unknown side lengths and angle measures.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The sine law can be applied when given particular measures of a triangle. Students may find a graphic organizer such as the following helpful:

Sine Law

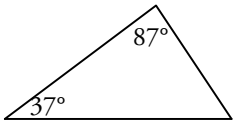
Angle-Side-Angle



10 cm

(ASA)

Angle-Angle-Side



10 cm

(AAS)

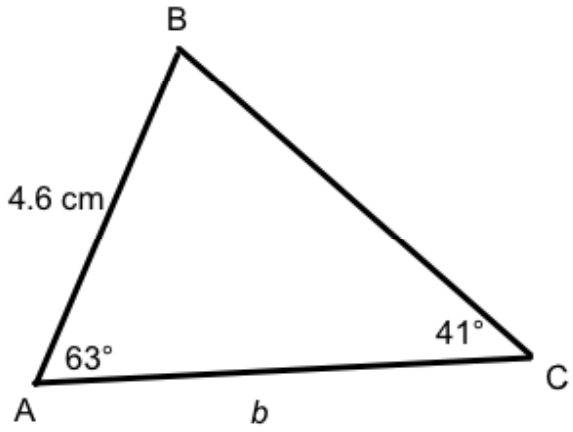
Students should realize that to use the sine law they may have to first determine the measure of the third angle.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Interview

- Ask students the following questions based on ΔABC:



(i) What information are you given in this triangle?
(ii) What are you trying to find in this triangle?
(iii) What other piece of information do you need to know in order to find the unknown measure?
(iv) How would you determine the unknown measure?

(G1.1)

Resources/Notes

Authorized Resource

Math at Work 12

7.1 The Sine Law

Student Book (SB): pp. 326-337

Teacher's Resource (TR): pp. 271-277

Note:

Explore the Sine Law on p. 326 is a good activity to introduce students to the sine law.

Geometry

Outcomes

Students will be expected to

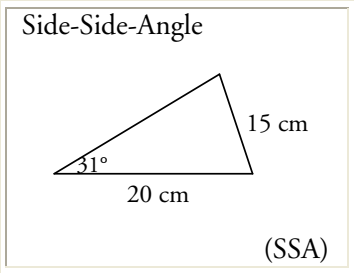
G1 Continued ...

Achievement Indicator:

G1.1 *Continued*

Elaborations—Strategies for Learning and Teaching

A third case, Side-Side-Angle, may give two possible angle measures, since it is the ambiguous case. Students are only required to determine one angle measure.



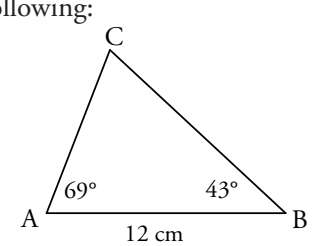
Before applying the sine law to determine unknown lengths and angle measures in triangles, ask students to consider what information is needed to solve problems using this law. The Law of Sines involves a ratio of the sine of an angle to the length of its opposite side. Students should recognize that it will not work if:

- no angle of the triangle is known
- one angle and its opposite side are not known.

The Law of Sines may be written as $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ or in its reciprocal form, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$. Using the reciprocal form could be beneficial for students when solving for an unknown angle.

Diagrams should be provided for all problems. Students should also be encouraged to re-draw diagrams with both the given and unknown information indicated.

When working with the sine law, students sometimes incorrectly identify side and opposite angle pairs. To avoid this error, encourage them to use arrows on the diagram when identifying the angle and its opposite side. Another common error occurs when students mistakenly think that, given the measures of two angles and the length of an included side, there is not enough information to use the sine law. Consider an example such as the following:



Students can use the property that the sum of the angles in a triangle is 180°. Therefore, the measure of ∠C is 68°. They can then proceed to use the sine law to find the length of side AC.

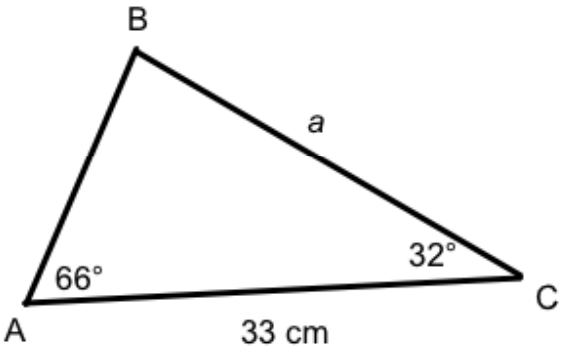
General Outcome: Develop spatial sense.

Suggested Assessment Strategies

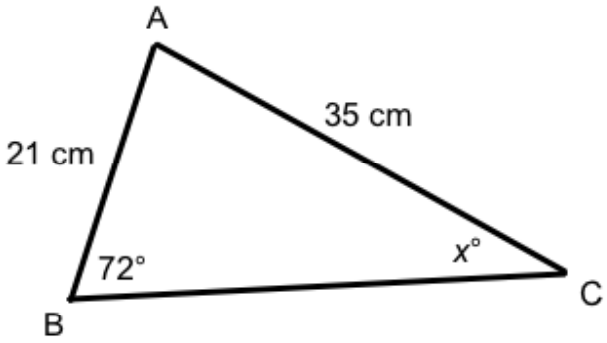
Paper and Pencil

- Ask students to use the sine law to find the missing measures.

(i)



(ii)



(G1.1)

Resources/Notes

Authorized Resource

Math at Work 12

7.1 The Sine Law

SB: pp. 326-337

TR: pp. 271-277

Geometry

Outcomes

Students will be expected to

G1 Continued ...

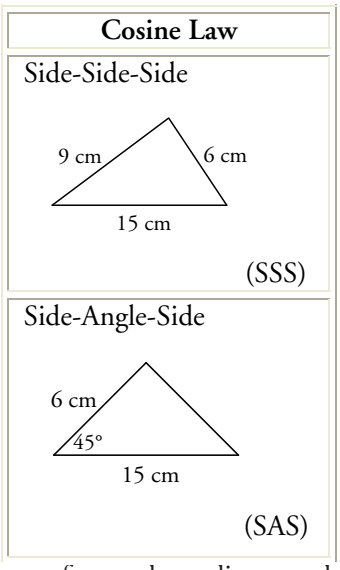
Achievement Indicator:

G1.2 *Solve a problem, using the cosine law, when a diagram is given.*

Elaborations—Strategies for Learning and Teaching

Students will apply the cosine law to determine unknown lengths and angle measures in triangles. They should be provided with diagrams for all problems, but also encouraged to re-draw diagrams with both the given and unknown information indicated.

The Law of Cosines is useful for determining the third side of a triangle when two sides and their enclosed angle are known, and in computing the angles of a triangle if all three sides are known. The following graphic organizer could be used:



The cosine law may be written in two forms, depending on whether the student is solving for a missing side or angle:

$$a^2 = b^2 + c^2 - 2bc\cos A \qquad \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

It is important that students recognize the patterns that exist in the cosine formula.

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Once the pattern is understood, they can write the cosine law for any labelled triangle.

They should consider why the cosine law is the only option to find the unknown angle if three sides are known or if two sides and the included angle are known. Common errors encountered by students using the cosine law may include:

- uisng the formula to determine a side when the missing measure is an angle
- not applying the order of operations correctly.

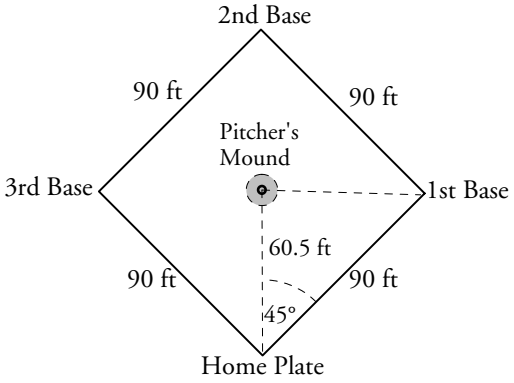
General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to answer the following:

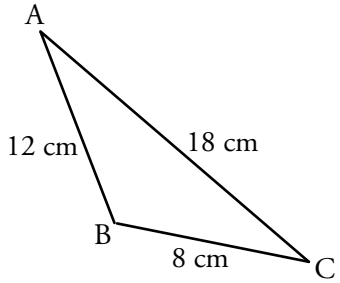
  The pitcher's mound on a baseball diamond is 60.5 ft from home plate. First base is 90 ft from home plate. Determine the distance the pitcher must throw the ball to get a runner out on first base.



(G1.2)

- Ask students to find the error(s) that was made to calculate $\angle C$ and provide the correct solution.

(i)



$\cos\angle C = \frac{a^2+b^2-c^2}{2ab}$

$\cos\angle C = \frac{8^2+12^2-18^2}{2(8)(18)}$ Step 1

$\cos\angle C = \frac{16+144-324}{288}$ Step 2

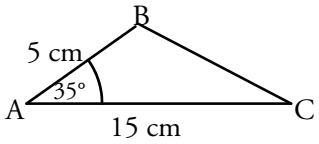
$\cos\angle C = \frac{-164}{288}$ Step 3

$\cos\angle C = -0.569$ Step 4

$\angle C = \cos^{-1}(-0.569)$ Step 5

$\angle C = 125°$ Step 6

(ii)



$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 15^2 + 5^2 - 2(15)(5)(\cos 35°)$ Step 1

$a^2 = 225 + 25 - 150(\cos 35°)$ Step 2

$a^2 = 100(\cos 35°)$ Step 3

$a^2 = 100(0.819)$ Step 4

$a^2 = 81.9$ Step 5

$a = 9.1$ Step 6

(G1.2)

Resources/Notes

Authorized Resource

Math at Work 12

7.2 The Cosine Law

SB: pp. 338-351

TR: pp. 278-285

Note:

Explore the Cosine Law on pp. 338-339 is a good activity to introduce students to the cosine law.

Geometry

Outcomes

Students will be expected to

G1 Continued ...

Achievement Indicator:

G1.3 *Solve a problem, using the sine law or cosine law, when a diagram is given.*

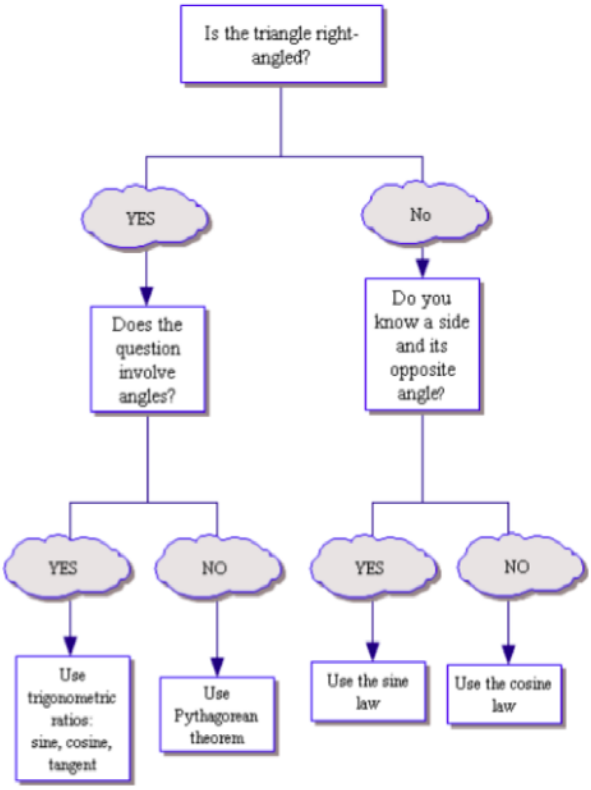
Elaborations—Strategies for Learning and Teaching

When solving triangles, encourage students to consider the following questions:

- What is the given information?
- What am I trying to solve for?
- With the given information, should I use the sine law or the cosine law? Is there a choice?
- Which form of the law do I use to solve for an unknown side? Which form do I use to solve for an unknown angle?

If students know two sides and a non-included angle, they can use the cosine law in conjunction with the sine law to find the other side. As an alternative, they could apply the sine law twice. Students should be exposed to numerous examples to find the method that works best for them.

Ask students to create a graphic organizer such as the following:



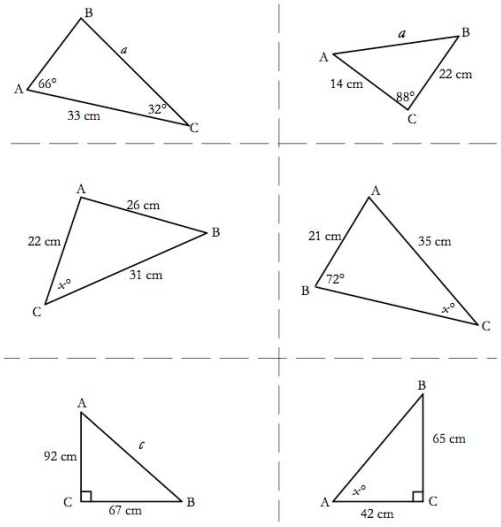
When solving triangles, the organizer can guide students as they decide on the most efficient method to use when solving for an unknown angle and/or side.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Performance

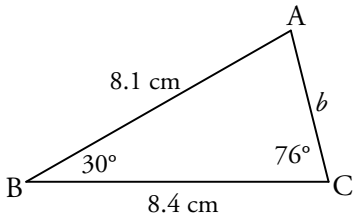
- In the activity *Four Corners*, students have to think about which method they would use to solve a triangle. Post four signs, one in each corner, labeled sine law, cosine law, Pythagorean theorem, trigonometric ratios. Provide each student with one triangle. Instruct the students to make a decision as to which method they would use to find the missing angle or side and to stand in the corner where it is labeled. Once students are all placed, ask them to discuss why their triangle(s) would be best solved using that particular method. Sample triangles are given below:



(G1.3)

Paper and Pencil

- Ask students to find the missing side length (b) using both the sine law and the cosine law.



They should then answer the following:

(i) What information was not needed to solve using the sine law?
(ii) What information was not needed to solve using the cosine law?
(iii) Which method was preferred and why?

(G1.3)

Resources/Notes

Authorized Resource

Math at Work 12

7.3 Solving Trigonometric Problems

SB: pp. 352-359

TR: pp. 286-291

Geometry

Outcomes	Elaborations—Strategies for Learning and Teaching
Students will be expected to **G1 Continued ...**	
Achievement Indicator: G1.4 *Describe the use of the sine law and cosine law in construction, industrial, commercial and artistic applications.*	Students should develop an appreciation for the applicability of the sine law and cosine law to real-life situations. This may be developed throughout the unit by using examples that are based on construction, industrial, commercial and artistic applications. Teachers may invite students to brainstorm other applications of the sine law and cosine law. Other examples may include aircraft design and flight. The aircraft engineer must calculate the plane's velocity as well as the air velocity in order to make it as aerodynamic as possible. The sine law can be used to find the angle that the plane must travel to compensate for wind velocity. The cosine law can be used to determine the magnitude of the resultant ground speed of the aircraft along the chosen bearing direction. Although students would not be expected to complete such a problem, teachers should expose students to a variety of applications.
N1 Analyze puzzles and games that involve logical reasoning, using problem-solving strategies. **[C, CN, PS, R]**	Students could play a murder mystery game. The students are the suspects and they try to solve the mystery by examing all of the clues in an attempt to identify the murderer. They work in small groups to answer questions using the sine law or the cosine law. With each correct answer, they are given a clue to help them solve the mystery. The clues should be designed to elicit increasingly more information about the murder, until the players are in a good position to suggest who they believe is the guilty party.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Presentation

- Ask students to research a real-life application of the sine law or cosine law. They should prepare a brief explanation of how the law applies to their selection and present their findings to the class.

(G1.4)

Resources/Notes

Authorized Resource

Math at Work 12

7.3 Solving Trigonometric Problems

SB: pp. 352-359

TR: pp. 286-291

Games and Puzzles

Jumping Beans

SB: p. 365

TR: p. 296

Appendix:

Outcomes with Achievement Indicators Organized by Topic (With Curriculum Guide Page References)

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Topic: Measurement	**General Outcome**: Develop spatial sense through direct and indirect measurement.	
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome.*	**Page Reference**
M1. Demonstrate an understanding of the limitations of measuring instruments, including: • precision • accuracy • uncertainty • tolerance and solve problems. [C, PS, R, T, V]	M1.1 Explain why, in a given context, a certain degree of precision is required.	p. 22
	M1.2 Explain why, in a certain context, a certain degree of accuracy is required.	p. 22
	M1.3 Explain, using examples, the difference between precision and accuracy.	p. 24
	M1.4 Analyze precision and accuracy in a contextual problem.	p. 24
	M1.5 Compare the degree of accuracy of two given instruments used to measure the same attribute.	p. 24
	M1.6 Describe, using examples, the limitations of measuring instruments used in a specific trade or industry.	p. 24
	M1.7 Relate the degree of precision to the uncertainty of a given measure.	p. 24
	M1.8 Calculate maximum and minimum values, using a given degree of tolerance in context.	pp. 24-26
	M1.9 Solve a problem that involves precision, accuracy or tolerance.	pp. 24-26

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Topic: Probability	**General Outcome**: Develop critical thinking skills related to uncertainty.	
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome.*	**Page Reference**
P1. Analyze and interpret problems that involve probability. [C, CN, PS, R]	P1.1 Calculate the probability of an event based on a data set.	pp. 30-32
	P1.2 Express a given probability as a fraction, decimal and percent and in a statement.	pp. 30-32
	P1.3 Explain the difference between odds and probability.	p. 32
	P1.4 Determine the probability of an event, given the odds for or against.	p. 32
	P1.5 Explain, using examples, how decisions may be based on a combination of theoretical probability calculations, experimental results and subjective judgements.	p. 34
	P1.6 Describe and explain the application of probability.	p. 34
	P1.7 Solve a contextual problem that involves odds or probability.	p. 34

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Topic: Number	**General Outcome**: Develop number sense and critical thinking skills.	
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome.*	**Page Reference**
N1. Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies. [C, CN, PS, R]	N1.1 Determine, explain and verify a strategy to solve a puzzle or to win a game; e.g., • guess and check • look for a pattern • make a systematic list • draw or model • eliminate possibilities • simplify the original problem • work backward • develop alternative approaches. N1.2 Identify and correct errors in a solution to a puzzle or in a strategy for winning a game. N1.3 Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.	pp. 38-47 pp. 70-71 pp. 84-85 pp. 98-99 pp. 118-119 pp. 134-135 pp. 148-149

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Topic: Number	**General Outcome**: Develop number sense and critical thinking skills.	
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome.*	**Page Reference**
N2. Solve problems that involve the acquisition of a vehicle by: • buying • leasing • leasing to buy. [C, CN, PS, R, T]	N2.1 Describe and explain various options for buying, leasing and leasing to buy a vehicle.	p. 90
	N2.2 Justify a decision related to buying, leasing or leasing to buy a vehicle, based on factors such as personal finances, intended use, maintenace, warranties, mileage and insurance.	p. 90
	N2.3 Solve, with or without technology, problems that involve the purchase, lease or lease to purchase a vehicle.	pp. 92-94
	N2.4 Determine costs of operating a vehicle, including fixed costs, such as extended warranty, insurance and licensing, and ongoing costs, such as gas and maintenance.	p. 94
N3. Critique the viability of small business options by considering: • expenses • sales • profit or loss. [C, CN, R]	N3.1 Identify expenses in operating a small business.	p. 96
	N3.2 Identify feasible small business options for a given community.	p. 96
	N3.3 Generate options that might improve the profitability of a small business.	p. 96
	N3.4 Determine the profit, loss and break-even point for a small business.	p. 98
	N3.5 Explain factors, such as seasonal variations and hours of operation, that might impact the profitability of a small business.	p. 98

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Topic: Statistics	**General Outcome**: Develop statistical reasoning.	
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome.*	**Page Reference**
S1. Solve problems that involve measures of central tendency, including: • mean • median • mode • weighted mean • trimmed mean. [C, CN, PS, R]	S1.1 Determine the mean, median and mode for a set of data.	pp. 52-54
	S1.2 Solve a contextual problem that involves measures of central tendency.	pp. 52-54
	S1.3 Identify and correct errors in a calculation of a measure of central tendency.	p. 56
	S1.4 Explain, using examples, the advantages and disadvantages of each measure of central tendency.	p. 58
	S1.5 Explain, using examples sich as course marks, why some data in a set would be given a greater weighting in determining the mean.	p. 60
	S1.6 Calculate the mean of a set of numbers after allowing the data to have different weightings (weighted mean).	p. 60
	S1.7 Identify the outlier(s) in a set of data.	p. 62
	S1.8 Explain the effect of outliers on mean, median and mode.	p. 62
	S1.9 Calculate the trimmed mean for a set of data, and justify the removal of the outliers.	p. 62
	S1.10 Explain, using examples from print and other media, how measures of central tendency and outliers are used to provide different interpretations of data.	p. 64

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Topic: Statistics	**General Outcome**: Develop statistical reasoning.	
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome.*	**Page Reference**
S2. Ananlyze and describe percentiles. [C, CN, PS, R]	S2.1 Explain the relationship between median and percentile.	p. 66
	S2.2 Explain, using examples, percentile ranks in a context.	p. 66
	S2.3 Explain, using examples, the difference between percent and percentile rank.	p. 66

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Topic: Algebra	**General Outcome**: Develop algebraic reasoning.	
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome.*	**Page Reference**
A1. Demonstrate an understanding of linear relations by: • recognizing patterns and trends • graphing • creating tables of values • writing equations • interpolating and extrapolating • solving problems. [CN, PS, R, T, V]	A1.1 Describe the trends in the graph of a data set, including scatter plots.	pp. 68
	A1.2 Solve a contextual problem that requires interpolation or extrapolation of information.	pp. 68, 76
	A1.3 Match given contexts with their corresponding graphs, and explain the reasoning.	pp. 68, 78
	A1.4 Create, with or without technology, a graph to represent a given or collected data set.	p. 76
	A1.5 Sort a set of scatter plots according to the trends represented (linear, nonlinear or no trend).	p. 78
	A1.6 Identify and describe the characteristics of a linear relation represented in a graph, table of values or number pattern.	p. 78
	A1.7 Sort a set of graphs, tables of values and/or number patterns into linear and nonlinear relations.	p. 78
	A1.8 Explain why the points should or should not be connected on the graph for a context.	pp. 80-82
	A1.9 Relate slope and rate of change to linear relations.	p. 82
	A1.10 Identify a linear equation as having a direct or partial variation relationship.	p. 82
	A1.11 Create a table of values for a given equation of a linear relation.	p. 84
	A1.12 Solve a contextual problem that involves the application of a formula for a linear relation.	p. 84
	A1.13 Write an equation for a given context, including direct or partial variation.	p. 84

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Topic: Geometry	**General Outcome**: Develop spatial sense.	
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome.*	**Page Reference**
G1. Solve problems by using the sine law and cosine law, excluding the ambiguous case. [CN, PS, V]	G1.1 Solve a problem using the sine law, when a diagram is given.	pp. 140-142
	G1.2 Solve a problem, using the cosine law, when a diagram is given.	p. 144
	G1.3 Solve a problem, using the sine law or cosine law, when a diagram is given.	p. 146
	G1.4 Describe the use of the sine law and cosine law in construction, industrial, commercial and artistic applications.	p. 148
G2. Solve problems that involve: • triangles • quadrilaterals • regular polygons. [C, CN, PS, V]	G2.1 Describe and illustrate angle properties of triangles.	p. 104
	G2.2 Describe and illustrate angle properties of quadrilaterals.	p. 106
	G2.3 Describe and illustrate angle properties of regular polygons.	pp. 108-110
	G2.4 Explain, using examples, why a given property does or does not apply to certain polygons.	p. 110
	G2.5 Describe and illustrate side length properties of triangles.	p. 112
	G2.6 Describe and illustrate properties of quadrilterals in terms of side lengths, diagonal lengths and angles of intersection.	p. 112
	G2.7 Describe and illustrate the diagonal properties of regular polygons.	p. 112
	G2.8 Describe and illustrate line symmetry in triangles, quadrilaterals amd regular polygons.	pp. 114-116
	G2.9 Identify adn explain an application of the properties of polygons in construction, industrial, commercial, domestic and artistic contexts.	p. 118
	G2.10 Solve a contextual problem that involves the application of the properties of polygons.	p. 118

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Topic: Geometry	**General Outcome**: Develop spatial sense.	
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome.*	**Page Reference**
G3. Demonstrate an understanding of transformations on a 2-D shape or a 3-D object, including: • translations • rotations • reflections • dilations. [C, CN, R, T, V]	G3.1 Determine and explain whether a given image is a dilation of another given shape, using the concept of similarity.	p. 124
	G3.2 Draw, with or without technology, a dilation image for a given 2-D shape or 3-D object, and explain how the original 2-D shape or 3-D object and its image are proportional.	p. 124-126
	G3.3 Identify and describe the applications of transformations in construction, industrial, commercial, domestic and artistic contexts.	pp. 126-132
	G3.4 Solve a contextual problem that involves transformations.	pp. 126-132
	G3.5 Draw the image of a 2-D shape that results from a given single transformation.	pp. 128-132
	G3.6 Identify a single transformation that was performed, given the original 2-D shape or 3-D object and its image.	pp. 128-132
	G3.7 Draw the image of a 2-D shape that results from a given combination of successive transformations.	pp. 128-132
	G3.8 Explain the relationship between reflections and lines or planes of symmetry.	p. 130
	G3.9 Create, analyze and describe designs, using translations, rotations and reflections in all four quadrants of coordinate grid.	pp. 130-132

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