Calculus 3208 Inverse Trig Unit 6 Winter 2020 Name:________________

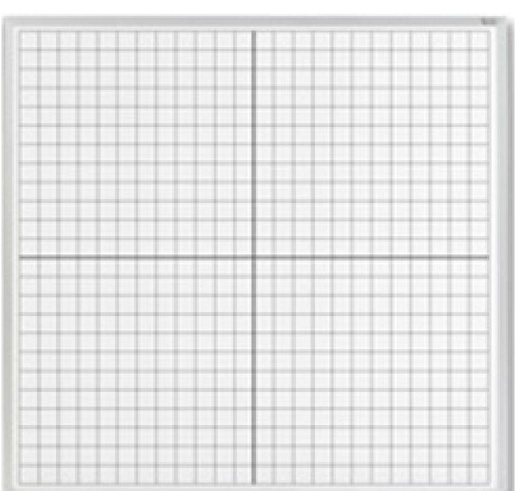
1 Evaluate the following limits knowing that

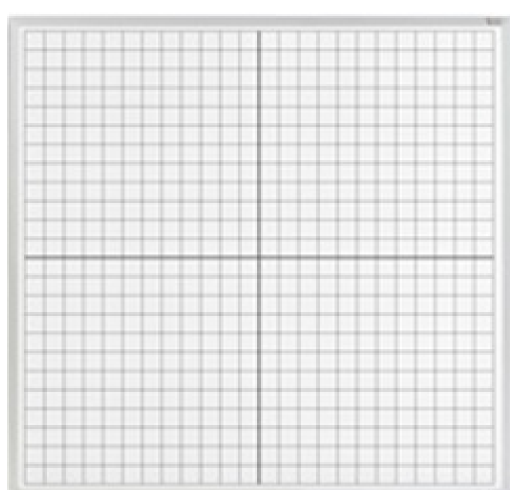
$\lim_{x \to 0} \sin x = 0$, $\lim_{x \to 0} \cos x = 1$, $\lim_{x \to 0} \frac{\sin x}{x} = 1$ , $\lim_{x \to 0} \frac{\cos x - 1}{x} = 0$

A) $\lim_{x \to 0} \frac{\sin(6x)}{x}$ B) $\lim_{x \to 0} \frac{\sin 8x}{2x}$ C) $\lim_{x \to 0} \frac{\sin 2x}{2x^2 + x}$

D) $\lim_{t \to 0} \frac{\tan(6t)}{t}$

2 Sketch the graphs of the following inverse trig functions be sure to state the domain, range (restriction) of each.



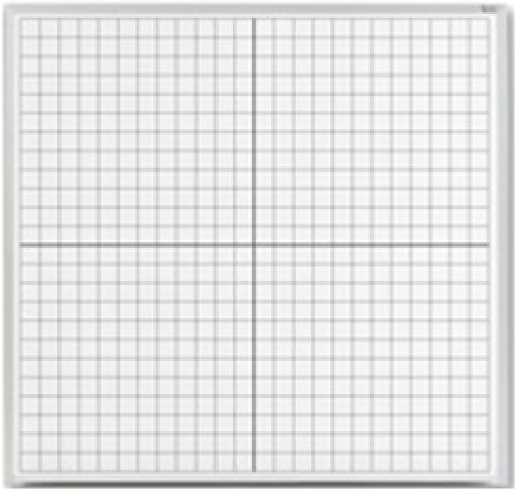


A) $y = \sin^{-1} x$ B) y=arccos x

Domain:

Range:

(State like $\frac{-\pi}{2} \le y \le \frac{\pi}{2}$ , $\frac{-\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2}$)



C) y = arctan x

Domain:

Range:

3 Prove that if f(x) = cos x then $f'(x) = -\sin x$ using the definition of the derivative. You will need the limits below and your sum-difference identity from m 3200.

Use $\lim_{x \to 0} \frac{\sin x}{x} = 1$ , $\lim_{x \to 0} \frac{\cos x - 1}{x} = 0$ $\quad f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

4 What is the equation of the tangent and normal line to the curve y = sin x at x = $\frac{3\pi}{4}$ ?

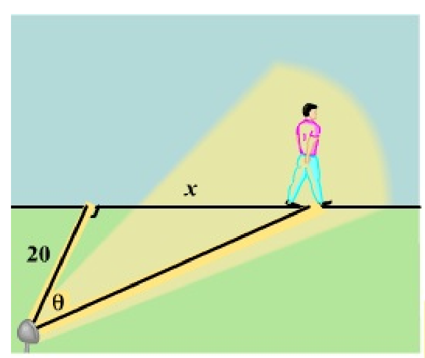
5 Determine the derivatives of the following:

$A)\ f(x) = \sec x \csc x \quad B)\ y = \sin(\sqrt{2x - 8x^2}) \quad C)\ y = \tan(\sqrt{4 - x^2}) \quad D)\ y = \frac{1 - \cos x}{\sin x}$

6 Determine $\frac{dy}{dx}$ for $x \sin(xy) = y \cos(4x)$ using implicit differentiation.

7 Find the equation of the tangent and normal lines to the curve $y=\sqrt{2}\csc x$ at x= $\frac{3\pi}{4}$.

8 A man walks along a straight path at a speed of 4 ft/s. A searchlight is located on the ground 20 ft from the path and is kept focused on the man. At what rate is the searchlight rotating when the man is 15 ft from the point on the path closest to the searchlight?




given: find _______ when x = _______

9 A particle moves along horizontally by the equation y = sint + sin 2t, where $0 \le x \le 2\pi$. If y is the displacement in meters and t is times in seconds, determine the absolute maximum and minimum displacements. (SCRAP!) Find dy/dx

10 Evaluate the following: (without the use of a calculator)

A) $\arccos\left(\frac{-1}{\sqrt{2}}\right)$ B) $\csc^{-1}(-2)$ C) $\arctan(-1)$

D) $\cos^{-1}(\sin\frac{\pi}{6})$ E) $\arctan(\cos\frac{3\pi}{2})$ F) $\sin^{-1}(\sin\frac{7\pi}{4})$

11 Find an expression(s) for the following in terms of x. Use Q1 only.

A) $\sec(\sin^{-1} x)$ B) $\tan(\cos^{-1} x)$ C) $\cot(\sec^{-1} x)$

12 Find without technology:

A) $\cos(\sin^{-1}(\frac{-1}{5}))$ B) $\cos^{-1}(\cos\frac{4\pi}{3})$ C) $\sin(\arcsin\frac{1}{\sqrt{2}} + arc\cos\frac{-1}{2})$

D) $\sin(2\sin^{-1}\frac{1}{3})$

13 Differentiate the following using the rules for inverse trig.

$A)\ \ y = \sin^{-1}(\sqrt{x+1})\ \ B)\ \ y = \tan^{-1}\sqrt{2x+1}\ \ \ C)\ \ y = \cos^{-1} x^2$

14 Find the slope of the tangent line of y = $\sin^{-1} x$ at x = .5 .

15 Find the equation of the tangent line to the curve $y = x\tan^{-1} x\ at\ x = 1$.

16 Show for the function y = x•arcsin(x) that

$$y^{||} = -\frac{x^2-2}{\sqrt{(1-x^2)^3}} \; or \; \frac{2-x^2}{\sqrt{(1-x^2)^3}}$$

.

End