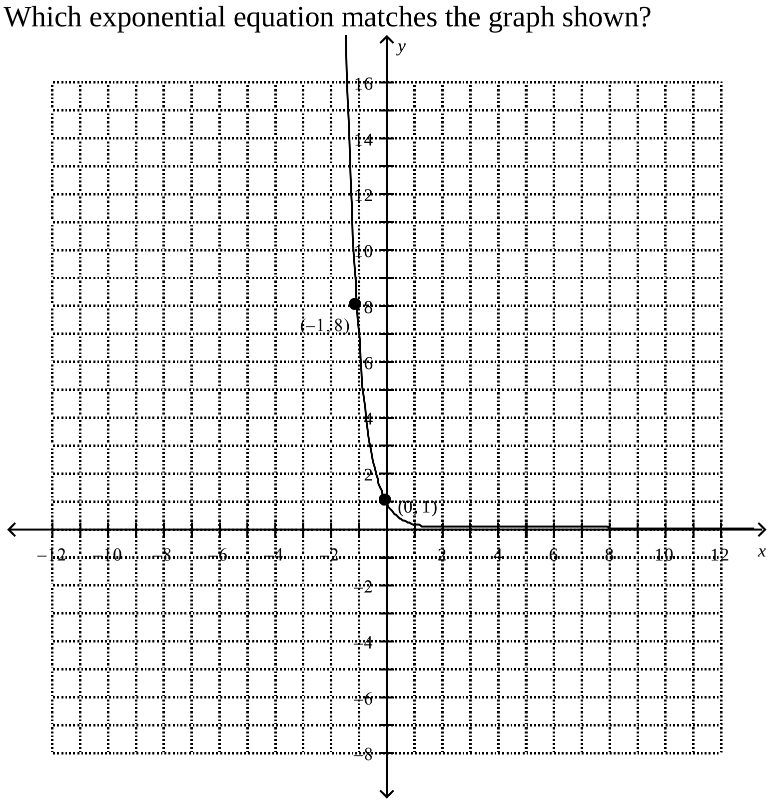
Assignment Unit 7 Winter 2020   Name:________________

Multiple Choice
Identify the choice that best completes the statement or answers the question.   1 mark each some are 2.

____ **1.** Which choice best describes the function $y = 20(\frac{1}{4})^{-x}$?

A both increasing and decreasing
B decreasing
C increasing
D neither increasing nor decreasing

____ **2.** Which exponential equation matches the graph shown?



A $y = \left(\frac{1}{8}\right)^x$
B $y = 8^x$
C $y = -\left(\frac{1}{8}\right)^x$
D $y = -8^x$

$2^{-x} = \left(\frac{1}{2}\right)^x$

____ **3.** A radioactive sample with an initial mass of 1 mg has a half-life of a 1/9 day. What is the equation that models the exponential decay, *A*, for time, *t*, ?

A $A = \left(\frac{1}{2}\right)^t$
B $A = \left(\frac{1}{2}\right)^{\frac{9}{t}}$
C $A = 2^{9t}$
D $A = 2^{\frac{9}{t}}$

____ **4.** A colony of ants has an initial population of 750 and triples every day. Which function can be used to model the ant population, *p*, after *t* days?

A $p(t) = 3(750)^t$
B $p(t) = \frac{1}{3}(750)^t$
C $p(t) = 750\left(\frac{1}{3}\right)^t$
D $p(t) = 750(3)^t$

____ **5.** A bacteria colony initially has 1500 cells and doubles every week. Which function can be used to model the population, *p*, of the colony after *t* days?

A $p(t) = 1500(3)^t$
B $p(t) = 1500(2)^t$
C $p(t) = 1500(2)^{\frac{t}{7}}$
D $p(t) = 1500(3)^{\frac{t}{7}}$

____ **6.** To the nearest year, how long would an investment need to be left in the bank at 5%, compounded annually, for the investment to triple?

A 15 years
B 26 years
C 28 years
D 23 years

____ **7.** Jennifer deposited some money into an account that pays 7% per year, compounded annually. Today her balance is $300. How much was in the account 10 years ago, to the nearest cent?

[Hint: Use $P = A(1+i)^{-n}$.]

A $163.18 **C** $42.86
B $30.00 **D** $152.50

____ **8.** For the exponential function, $y = 200(4)^{-x}$ which of the following statements is *not* true?

A The graph of the function is increasing.
B The graph of the function is decreasing.
C The domain is the set of real numbers.
D The range is the set of real numbers greater than zero.

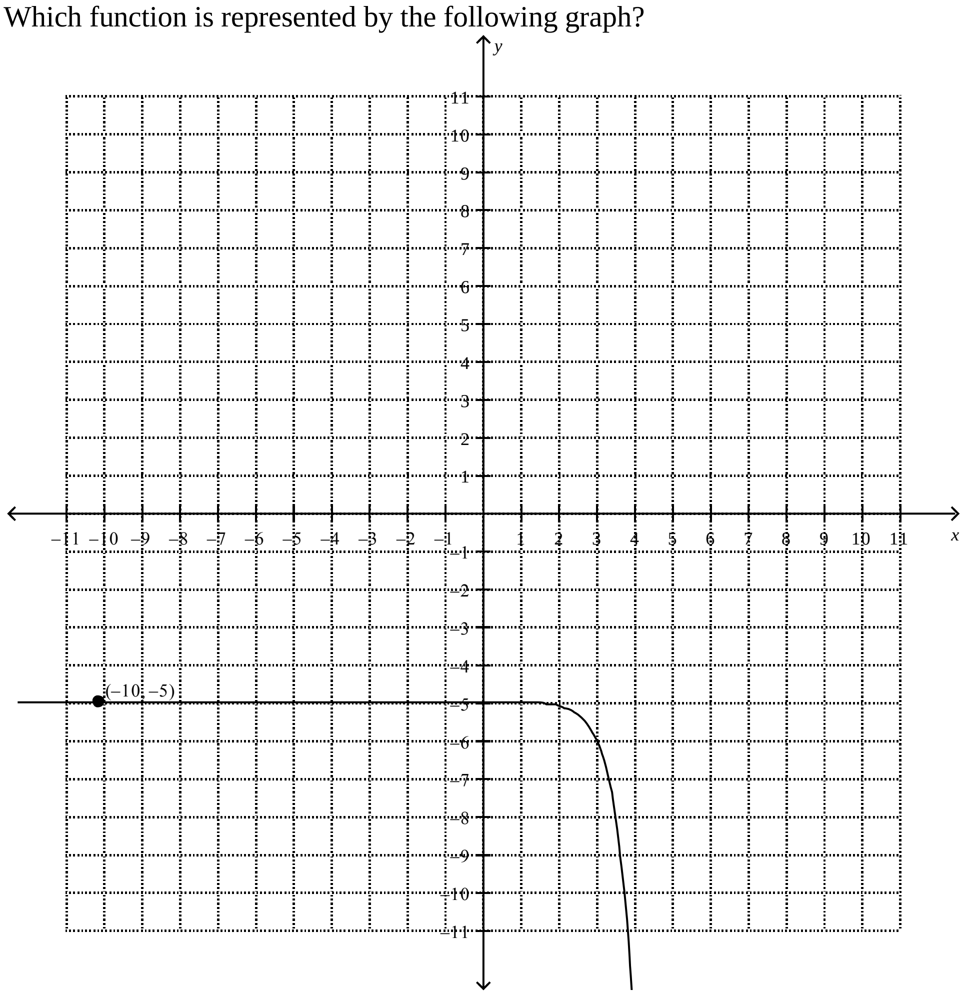
____ **9.** Which of the following transformations maps the function $y = 8^x$ onto the function $y = 8^{x+5} + 7$?

A a horizontal shift of 5 units to the left and a vertical shift of 7 units up
B a horizontal shift of 5 units to the right and a vertical shift of 7 units down
C a horizontal shift of 5 units to the right and a vertical shift of 7 units up
D a horizontal shift of 5 units to the left and a vertical shift of 7 units down

____ **10.** Which function results when the graph of $y = 6^x$ is translated 2 units down?

A $y = 6^{x-2}$ **C** $y = 6^x - 2$
B $y = 6^{x+2}$ **D** $y = 6^x + 2$

____ **11.** Which function is represented by the following graph?



A $y = -9(9)^{(x+4)} + 5$ **C** $y = -9(9)^{(x-4)} - 5$
B $y = 9(9)^{-(x+4)} + 5$ **D** $y = 9(9)^{-(x-4)} - 5$

____ **12.** Which function results when the graph of the function $y = 4^x$ is reflected in the *y*-axis, compressed vertically by a factor of $\frac{1}{5}$, and shifted 2 units down?

A $y = \frac{1}{5}(4)^{-x} - 2$ **C** $y = \frac{1}{5}(4)^{x} + 2$
B $y = \frac{1}{5}(4)^{x} - 2$ **D** $y = \frac{1}{5}(4)^{-x} + 2$

____ **13.** What is the exponential equation for the function that results from the transformations listed being applied to the base function $y = 9^x$?
• a reflection in the *y*-axis
• a vertical stretch by a factor of 6
• a horizontal stretch by a factor of 7

A $y = -7(9)^{\frac{x}{6}}$
C $y = 7(9)^{\frac{x}{6}}$
B $y = 6(9)^{\frac{-x}{7}}$
D $y = -6(9)^{\frac{x}{7}}$

____ **14.** Which equation can be used to model the given information, where the population has been rounded to the nearest whole number?

Year (*x*)	**Population (*y*)**
0	100
1	104
2	108
3	112
4	117
5	122

A $y = 100(1.04)^x$
C $y = 100(1.04)^{x-1}$
B $y = 100(1.4)^x$
D $y = 100(1.4)^{x-1}$

____ **15.** Solve for *x*. (Show workings!)

$1\,562\,500 = 4(5)^x$

A 9
C 8
B 7
D 11

____ **16.** Solve for *x*, to one decimal place. Calculator Logarithms required.

$7333 = 5^x$

A 1466.6
C 36 667.0
B 11.1
D 5.5

____ **17.** Solve for *x*. *Show workings!*

$(36)^{3x} = 216^{(x+7)}$

A 0.3
C 6
B 7
D 3.0

____ **18.** The half-life of a radioactive element can be modelled by $M = M_0\left(\frac{1}{32}\right)^{\frac{t}{45}}$, where M_0 is the initial mass of the element; t is the elapsed time, in hours; and M is the mass that remains after time t. The half-life of the element is

A 32 h
C 45 h
B 10 h
D 90 h

$m = m_0\left[\left(\frac{1}{2}\right)^5\right]^{\frac{t}{45}}$

_____19. **What is the value of e^4 to the nearest thousand?** **19**____________

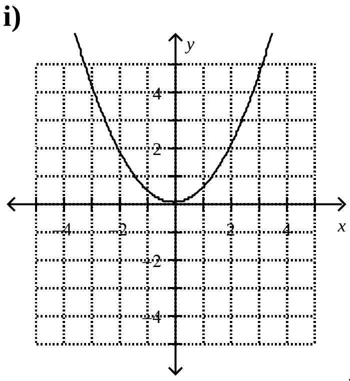
_____20. **Solve for x to the nearest one hundredth?** $e^{x+1} = 3056.421$ **20.**___________

_____21. **Compute 6e – 7e + e.** **21.**____________

Short Answer

1. a) Determine the type of function shown in each graph. 3 marks

i)



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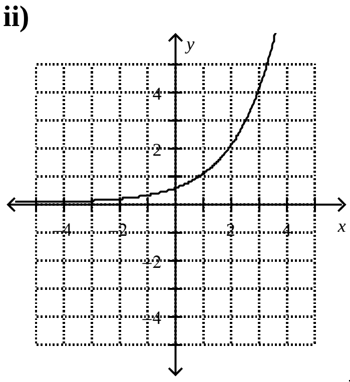
ii)



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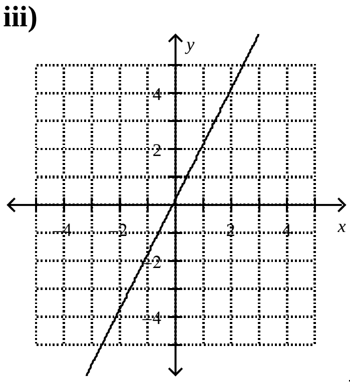
iii)



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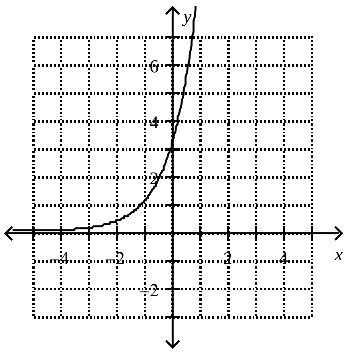
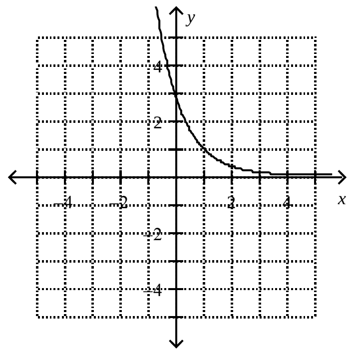
2. Match each graph with the correct corresponding equation. 4 marks

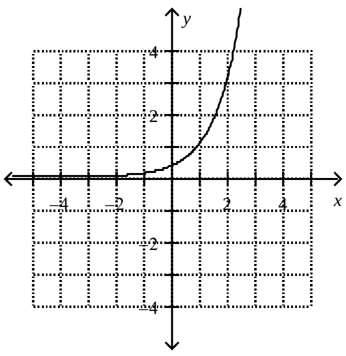
a)



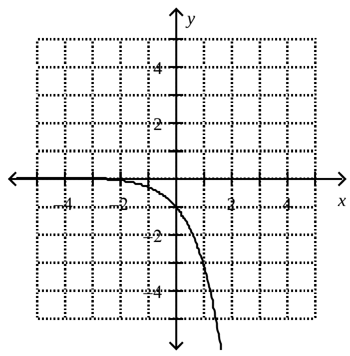
b)



c)



d)



i) $y = \frac{1}{3}(3)^x$

ii) $y = 3(3)^x$

iii) $y = -3^x$

iv) $y = 3\left(\frac{1}{3}\right)^x$

3. For the function, $y = 2(6)^{-3x-9} + 13$ 8 marks

a) describe the transformations of the function when compared to the function $y = 6^x$

VS

HS

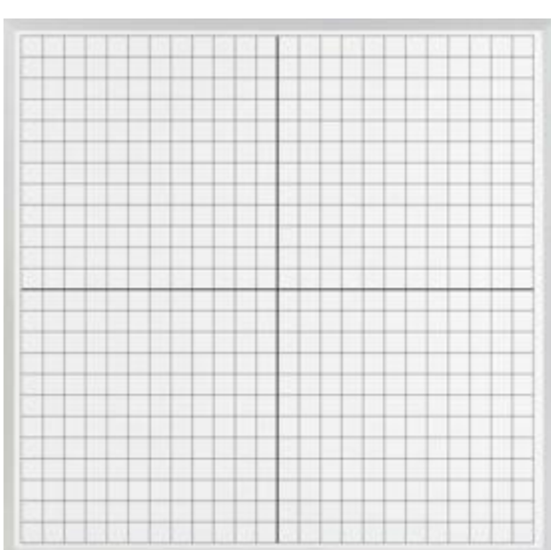
VT

HT

Reflection(s):

Mapping Rule:

b) sketch the graph of the function $y = 2^x$ and $y = -2^{2x+4} + 5$ on the same set of axes using the mapping rule and a table of values for both functions. 6 marks



c) state the domain, the range in interval notation and the equation of the asymptote for each in 3b: 3 marks

Domain: Range: Asymptote:

4. Write the equation for the function that results from each transformation or set of transformations applied to the base function $y = (1.5)^x$. 6 marks

a) reflect in the x-axis

b) shift 12 units to the left

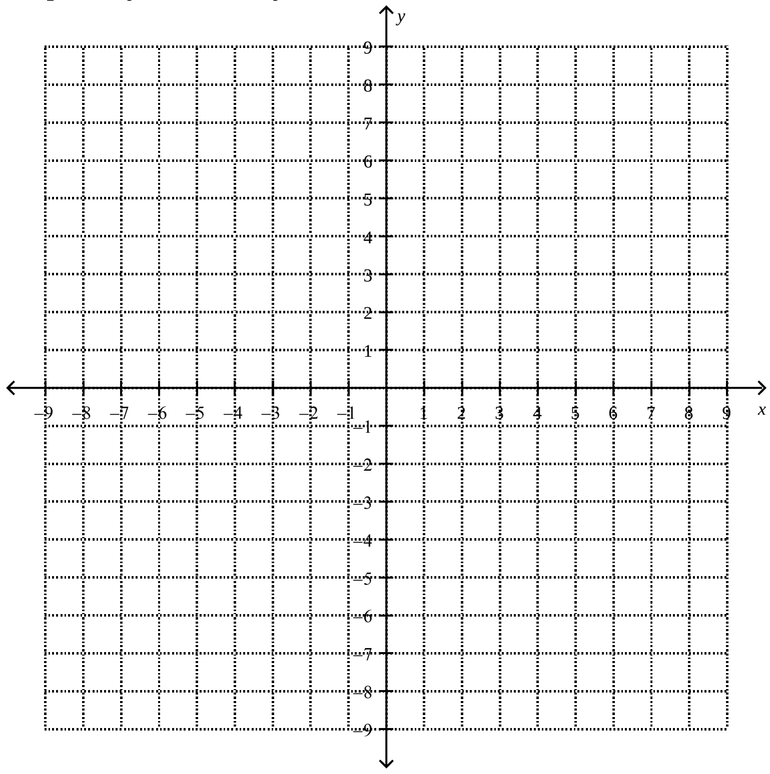
c) shift 10 unit down and 14 units to the left

d) reflect in the *x*-axis and shift 12 units down

5. Solve for *n*: $9^{x-1} = \left(\frac{1}{3}\right)^{4x-1}$ 3 marks

6. Solve for *x*:A) $3^x = 9^{x^2 - \frac{1}{2}}$ B) $\sqrt[6]{2} = \left(\frac{1}{64}\right)^{-x-3}$ C) $(3\sqrt{3})^x = 27^{2x+1}$ 16 marks

7. Graphically solve the system $y = 3^x$, $3x - y = 0$. 6 marks



Applications: For each application, a MODEL must be set up and then used to solve an exponential equation or expression that follows.

1. A colony of ants starts with an initial population of 100 and doubles every fourth week for 16 weeks.

a) Create a table of values for weeks 0 to 16 for the population of the colony. 2 marks

b) Is the relationship between the ant population and the number of weeks exponential? Explain. 1 mark

C) Model the information using an equation. 2 marks

D) Using C, algebraically determine how long it will take the colony to reach a population of 6400. 4 marks

2. Jeff buys a new vehicle for $65 000. It is known that the vehicle will depreciate by 24% of its current value every year. 8 marks

a) Write an equation to relate the depreciated value, *V*, of the vehicle to the age, *t*, in years, of the vehicle.

b) Use the equation to determine the value of the vehicle 3 years after Jeff buys it.

c) Approximately how long will it take the vehicle to depreciate to $15,000? (Use TI-83)

3. Cobalt-60, which has a half-life of 5.25 years, is used in medical radiology. A sample of 200 mg of the material is present today. 20 marks

a) Write an equation to relate the amount of cobalt-60 remaining and the number of half-life periods.

b) What amount will be present in 12.6 years to one decimal place?

c) Algebraically, how many years will it take for the amount of cobalt-60 to decay to one quarter of its initial amount?

d) Algebraically, how long will it take to decay to 12.5% of its original amount?

e) Algebraically, determine how long it will take to decay to 3.125 mg?

4. A radioactive sample with an initial mass of 72 mg has a half-life of 10 days. 8 marks

a) Write a function to relate the amount remaining, *A*, in milligrams, to the time, *t*, in days.

b) What amount of the radioactive sample will remain after 20 days?

c) Algebraically determine how long it will take to decay to 9 mg?

5. Solve the equation $\sqrt[3]{256^2} \times 16^x = 64^{x-3}$. 6 marks

7. An $8000 investment is being made for 10 years in to a GIC. Set up a model for each investment below and determine which investment is best. 9 marks

A) 4% compounded semi-annually B) 3% compounded quarterly

Model: Model:

Solution: Solution:

Conclusion:

End: Exam Date:_________________

END

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Answer Section

MULTIPLE CHOICE

1. ANS: C PTS: 1 DIF: Easy OBJ: Section 7.1
NAT: RF9 TOP: Characteristics of Exponential Functions
KEY: increasing | decreasing

2. ANS: A PTS: 1 DIF: Average OBJ: Section 7.1
NAT: RF9 TOP: Characteristics of Exponential Functions
KEY: equation | graph | exponential function

3. ANS: A PTS: 1 DIF: Average OBJ: Section 7.1
NAT: RF9 TOP: Characteristics of Exponential Functions
KEY: modelling | exponential decay

4. ANS: D PTS: 1 DIF: Easy OBJ: Section 7.2
NAT: RF9 TOP: Transformations of Exponential Functions
KEY: modelling | exponential growth

5. ANS: C PTS: 1 DIF: Average OBJ: Section 7.2
NAT: RF9 TOP: Transformations of Exponential Functions
KEY: modelling | exponential growth

6. ANS: D PTS: 1 DIF: Easy OBJ: Section 7.3
NAT: RF10 TOP: Solving Exponential Equations KEY: compound interest

7. ANS: D PTS: 1 DIF: Average OBJ: Section 7.2
NAT: RF10 TOP: Transformations of Exponential Functions
KEY: modelling | exponential decay

8. ANS: A PTS: 1 DIF: Easy OBJ: Section 7.1 | Section 7.2
NAT: RF9
TOP: Characteristics of Exponential Functions | Transformations of Exponential Functions
KEY: increasing | decreasing | domain | range

9. ANS: A PTS: 1 DIF: Average OBJ: Section 7.2
NAT: RF9 TOP: Transformations of Exponential Functions
KEY: transformations of exponential functions

10. ANS: C PTS: 1 DIF: Easy OBJ: Section 7.2
NAT: RF9 TOP: Transformations of Exponential Functions
KEY: transformations of exponential functions

11. ANS: C PTS: 1 DIF: Difficult OBJ: Section 7.2
NAT: RF9 TOP: Transformations of Exponential Functions
KEY: transformations of exponential functions

12. ANS: A PTS: 1 DIF: Average OBJ: Section 7.2
NAT: RF9 TOP: Transformations of Exponential Functions
KEY: transformations of exponential functions

13. ANS: B PTS: 1 DIF: Easy OBJ: Section 7.2
NAT: RF9 TOP: Transformations of Exponential Functions
KEY: transformations of exponential functions

14. ANS: A PTS: 1 DIF: Difficult OBJ: Section 7.1
NAT: RF9 TOP: Characteristics of Exponential Functions
KEY: modelling | exponential function

15. ANS: C PTS: 1 DIF: Easy OBJ: Section 7.3
NAT: RF10 TOP: Solving Exponential Equations KEY: change of base

16. ANS: D PTS: 1 DIF: Average OBJ: Section 7.3
NAT: RF10 TOP: Solving Exponential Equations
KEY: exponential equation | systematic trial

17. ANS: B PTS: 1 DIF: Average OBJ: Section 7.3
NAT: RF10 TOP: Solving Exponential Equations
KEY: exponential equation | equate exponents

18. ANS: D PTS: 1 DIF: Difficult OBJ: Section 7.3
NAT: RF10 TOP: Solving Exponential Equations KEY: half-life | exponential decay

SHORT ANSWER

1. ANS:
a) i) quadratic
ii) exponential
iii) linear
b) i) successive values would be increasing by a constant amount
ii) successive values would be increasing by a constant factor
iii) all values would be constant

PTS: 1 DIF: Average OBJ: Section 7.1 NAT: RF9
TOP: Characteristics of Exponential Functions
KEY: linear | quadratic | exponential function

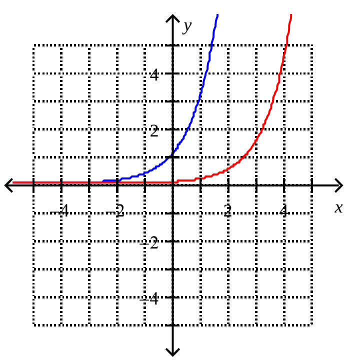
2. ANS:
a) ii)
b) iv)
c) i)
d) iii)

PTS: 1 DIF: Easy OBJ: Section 7.1 NAT: RF9
TOP: Characteristics of Exponential Functions
KEY: equation | graph | exponential function

3. ANS:

a) a vertical compression by a factor of $\frac{1}{2}$ and a translation of 2 units to the right

b) The graph of $y = 3^x$ is shown in blue and the graph of $y = \frac{1}{2}(3)^{x-2}$ is shown in red.



c) domain $\{x|x \in \mathrm{R}\}$, range $\{y|y > 0, y \in \mathrm{R}\}$, $y = 0$

PTS: 1 DIF: Average OBJ: Section 7.2 NAT: RF9
TOP: Transformations of Exponential Functions
KEY: graph | transformations of exponential functions

4. ANS:

a) $y = 5^{-x}$

b) $y = 5^{x-3}$

c) $y = 5^{x+4} - 1$

d) $y = -5^{x} - 2$

PTS: 1 DIF: Average OBJ: Section 7.2 NAT: RF9
TOP: Transformations of Exponential Functions
KEY: equation | transformations of exponential functions

5. ANS:

$$9^{n-1} = \left(\frac{1}{3}\right)^{4n-1}$$

$$\left(3^2\right)^{n-1} = \left(3^{-1}\right)^{4n-1}$$

$$3^{2n-2} = 3^{1-4n}$$

Equate the exponents:

$$2n - 2 = 1 - 4n$$

$$6n = 3$$

$$n = \frac{1}{2}$$

PTS: 1 DIF: Average OBJ: Section 7.3 NAT: RF10
TOP: Solving Exponential Equations KEY: change of base

6. ANS:

$$3^x = 9^{x^2 - \frac{1}{2}}$$

$$3^x = 3^{2\left(x^2 - \frac{1}{2}\right)}$$

Equate the exponents:

$$x = 2x^2 - 1$$

$$0 = 2x^2 - x - 1$$

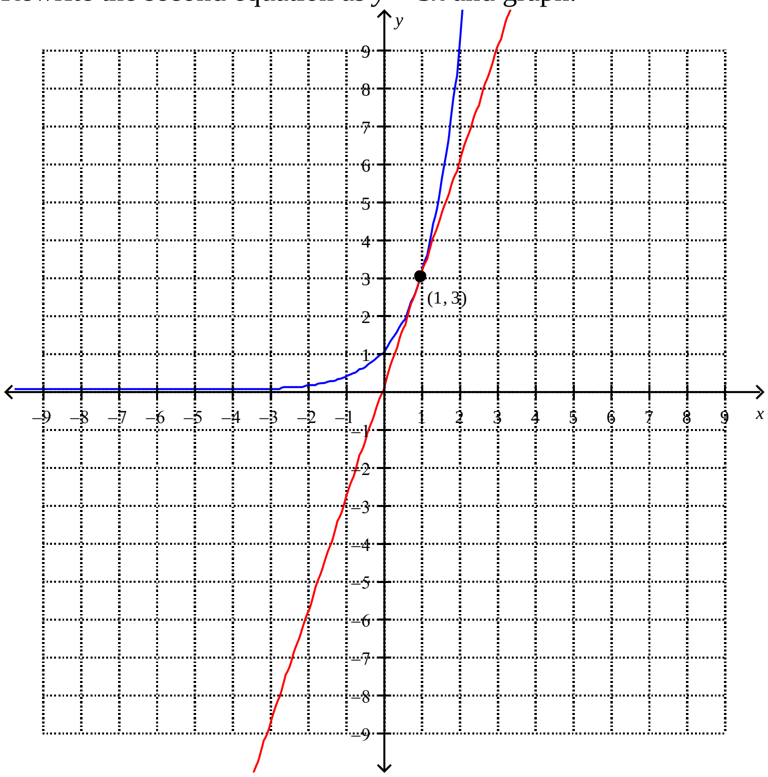
$$0 = (2x + 1)(x - 1)$$

$$x = -\frac{1}{2}, \qquad x = 1$$

PTS: 1 DIF: Difficult OBJ: Section 7.3 NAT: RF10
TOP: Solving Exponential Equations KEY: change of base | equate exponents

7. ANS:

Rewrite the second equation as $y = 3x$ and graph:



The solution is (1, 3).

PTS: 1 DIF: Difficult + OBJ: Section 7.3 NAT: RF10
TOP: Solving Exponential Equations KEY: system of equations | solve by graphing

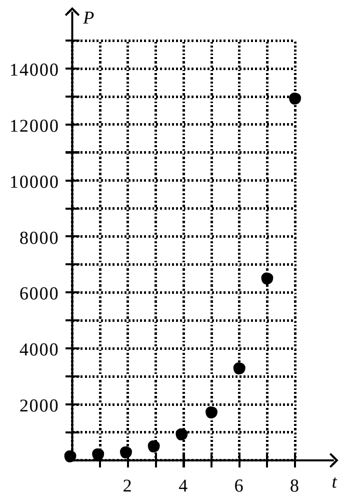
PROBLEM

1. ANS:

a)

Time, *t* (weeks)	**Population, *P***
0	50
1	100
2	200
3	400
4	800
5	1 600
6	3 200
7	6 400
8	12 8

b)



c) The data seem to be exponential, since the graph increases at an increasing rate. The values for population are being multiplied by a factor of 2 between successive terms in the table of values.
d) $P = 50(2)^t$

PTS: 1 DIF: Average OBJ: Section 7.1 NAT: RF9
TOP: Characteristics of Exponential Functions
KEY: graph | modelling | exponential growth

2. ANS:

a) $V = 35\ 000(0.80)^t$
b) $V = 35\ 000(0.80)^t$

$= 35\ 000(0.80)^2$

$= 22\ 400$

The value of the vehicle after 2 years is $22 400.

c) $V = 35\ 000(0.80)^t$

$3000 = 35\ 000(0.80)^t$

Use systematic trial. When $t = 11$, $V = 3006.48$. Therefore, after approximately 11 years, the vehicle will be worth $3000.

PTS: 1 DIF: Average OBJ: Section 7.3 NAT: RF10
TOP: Solving Exponential Equations KEY: modelling | exponential decay | systematic trial

3. ANS:

a) $A = 60\left(\frac{1}{2}\right)^n$, where *A* is the amount of cobalt-60 remaining, in milligrams, and *n* is the number of half-life periods.
b) 10.6 years equals 2 half-life periods, since $5.3 \times 2 = 10.6$.

$$A = 60\left(\frac{1}{2}\right)^{n}$$

$$= 60\left(\frac{1}{2}\right)^{2}$$

$$= \frac{60}{4}$$

$$= 15$$

15 mg will be present in 10.6 years.

c) $12.5\% = 0.125$

$$= \frac{1}{8}$$

$$\frac{1}{8} = \left(\frac{1}{2}\right)^{n}$$

$$\left(\frac{1}{2}\right)^{3} = \left(\frac{1}{2}\right)^{n}$$

$$3 = n$$

It will take 5.3 × 3, or 15.9 years, for the amount of cobalt-60 to decay to 12.5% of its initial amount.

PTS: 1 DIF: Difficult OBJ: Section 7.1 | Section 7.3
NAT: RF9 | RF10 TOP: Characteristics of Exponential Functions | Solving Exponential Equations
KEY: modelling | exponential decay | change of base

4. ANS:

a) $A = 6000$, $i = 0.035$, $n = 4$

b) $P = A(1 + i)^{-n}$

$$= 6000(1.035)^{-4}$$

$$\approx 5228.65$$

Therefore, she needs to invest $5228.65.

c) $P = A(1 + i)^{-n}$

$$= 6000(1.04)^{-4}$$

$$\approx 5128.83$$

Therefore, if the financial institution were to offer 4% annual interest, she would be able to invest approximately $100 less to have the same accumulated amount at the end of 4 years.

PTS: 1 DIF: Average OBJ: Section 7.3 NAT: RF10
TOP: Solving Exponential Equations KEY: modelling | exponential decay | negative exponents

5. ANS:

a) $A = 72\left(\frac{1}{2}\right)^{\frac{t}{10}}$

b) $A = 72\left(\frac{1}{2}\right)^{\frac{t}{10}}$

$= 72\left(\frac{1}{2}\right)^{\frac{20}{10}}$

$= 72\left(\frac{1}{2}\right)^{2}$

$= 18$

There will be 18 mg remaining after 20 days.

c) $A = 72\left(\frac{1}{2}\right)^{\frac{t}{10}}$

$= 72\left(\frac{1}{2}\right)^{\frac{-30}{10}}$

$= 72\left(\frac{1}{2}\right)^{-3}$

$= 576$

There was 576 mg 30 days ago.

d) $A = 72\left(\frac{1}{2}\right)^{\frac{t}{10}}$

$0.07 = 72\left(\frac{1}{2}\right)^{\frac{t}{10}}$

$\frac{0.07}{72} = \left(\frac{1}{2}\right)^{\frac{t}{10}}$

Use systematic trial.

$\frac{0.07}{72} \doteq 0.000\,972$

For t = 100, $\left(\frac{1}{2}\right)^{10} \doteq 0.000\,977$.

It will take approximately 100 days for there to be 0.07 mg remaining.

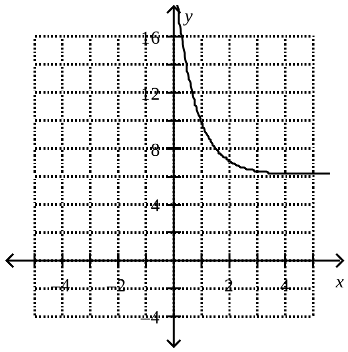
PTS: 1 DIF: Average OBJ: Section 7.2 | Section 7.3
NAT: RF9 | RF10 TOP: Transformations of Exponential Functions | Solving Exponential Equations
KEY: modelling | evaluate exponential functions

6. ANS:

a) $y = 2^{-2(x-2)} + 6$

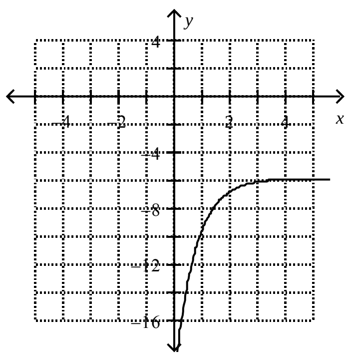
b) Reflect in the *y*-axis, compress horizontally by a factor of $\frac{1}{2}$, and translate 2 units to the right and 6 units up.

c)



d) $y = -2^{-2(x-2)} - 6$

e)



PTS: 1 DIF: Average OBJ: Section 7.2 NAT: RF9
TOP: Exponential Functions KEY: graph | transformations of exponential functions

7. ANS:

$$\sqrt[3]{256^2} \times 16^x = 64^{x-3}$$

$$(2^8)^{\frac{2}{3}} \times 2^{4x} = 2^{6x-18}$$

$$2^{4x+\frac{16}{3}} = 2^{6x-18}$$

$$4x + \frac{16}{3} = 6x - 18$$

$$-2x = -\frac{70}{3}$$

$$x = \frac{35}{3}$$

PTS: 1 DIF: Average OBJ: Section 7.3 NAT: RF10
TOP: Solving Exponential Equations KEY: exponential equation | change of base