

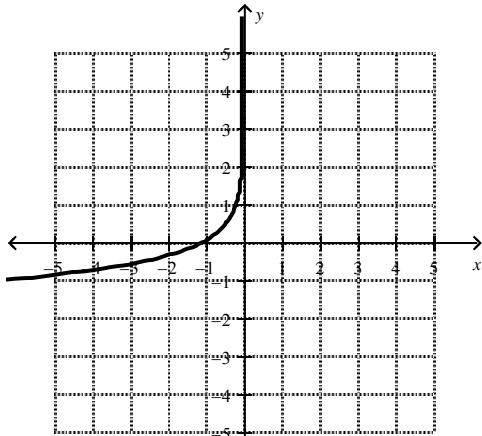
Assignment Unit 8 Logarithms May 2020 Name: _____

Multiple Choice

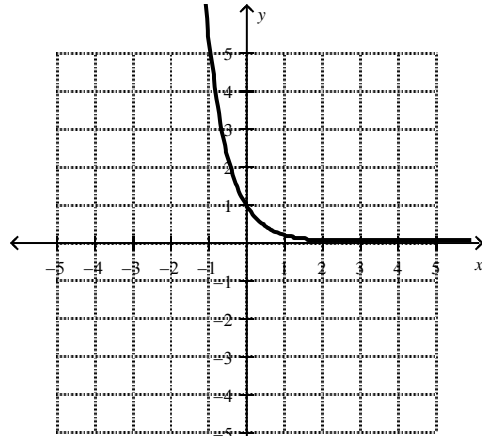
Identify the choice that best completes the statement or answers the question.

_____ 1. Which graph represents the inverse of $y = \left(\frac{1}{6}\right)^x$?

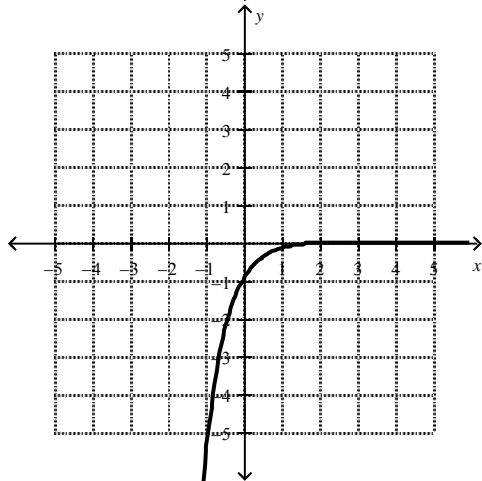
A.



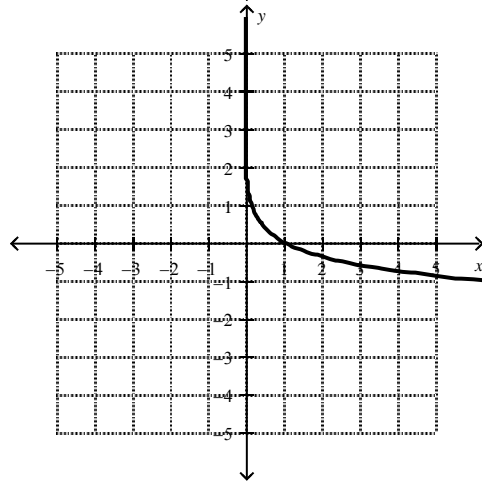
C.



B.



D.



_____ 2. Another way of writing $5^5 = 3125$ is

A. $\log_5 5 = 3125$

C. $\log_{3125} 5 = 5$

B. $\log_5 3125 = 5$

D. $\log_5 5 = 3125$

_____ 3. Which of the following represents $a = \log_7 343$?

A. $7^{343} = a$

C. $a^7 = 343$

B. $7^a = 343$

D. $a^{343} = 7$

_____ 4. Evaluate $\log_4 65536$.

A. 4096

C. 0.13

B. 5.33

D. 8

_____ 5. The domain of the function $f(x) = 8 \log_6 [8(x+8)] + 7$ is

A. $\{x | x < 7, x \in \mathbb{R}\}$

C. $\{x | x > 7, x \in \mathbb{R}\}$

B. $\{x | x < -8, x \in \mathbb{R}\}$

D. $\{x | x > -8, x \in \mathbb{R}\}$

_____ 6. Compared to the graph of the base function $y = \log_7 x$, the graph of the function $y = \log_7(x-6) + 7$ is translated

A. 6 units to the right and 7 units up

C. 7 units to the right and 6 units up

B. 7 units to the left and 6 units down

D. 6 units to the left and 7 units down

_____ 7. Compared to the graph of the base function $f(x) = \log_9 x$, the graph of the function $g(x) = 3 \log_9 [3(x+6)] - 10$ is

A. translated down 10 units and left 6 units, horizontally stretched by a factor of $\frac{1}{3}$,

C. translated down 10 units and left 6 units, horizontally stretched by a factor of $\frac{1}{3}$,

reflected in the y-axis, vertically stretched by a factor of 3, and reflected in the x-axis

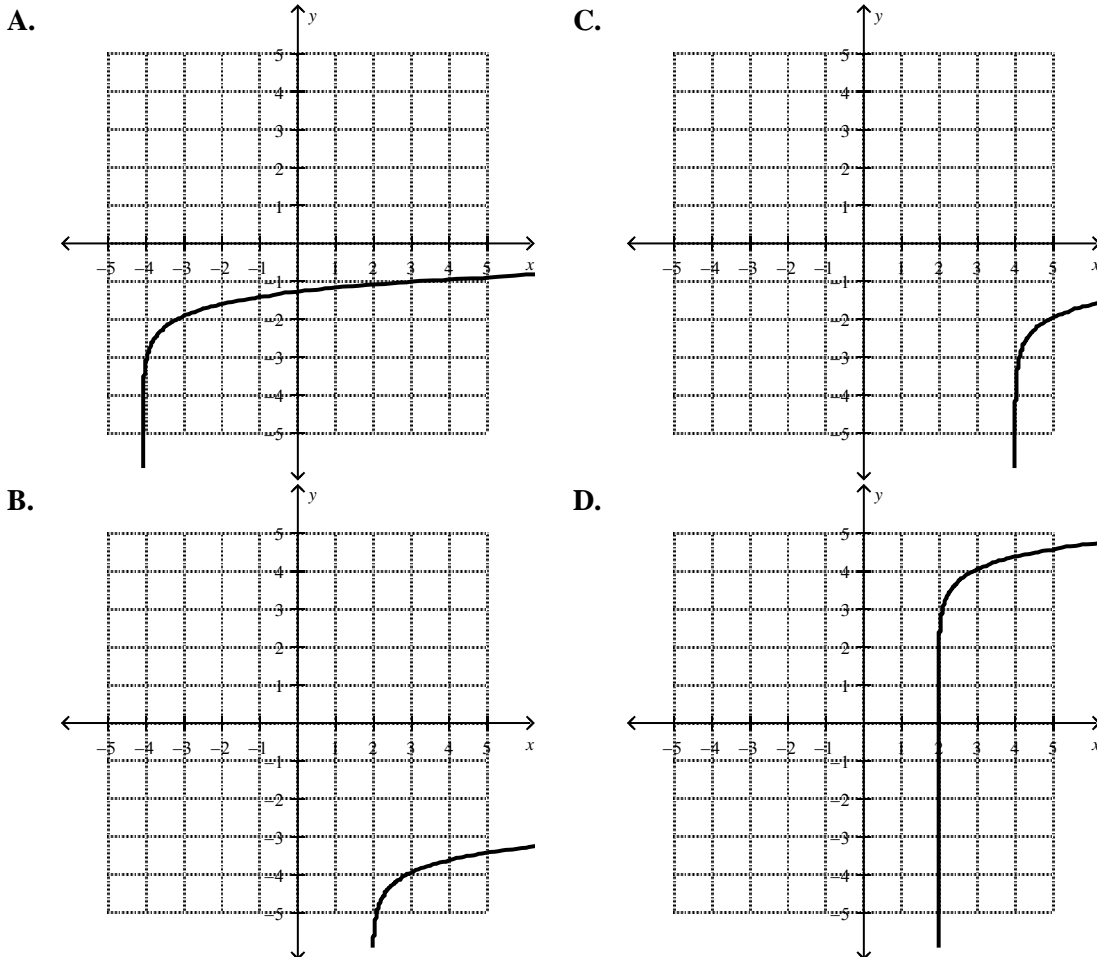
not reflected in the y-axis, vertically stretched by a factor of 3, and not reflected in the x-axis

- B.** translated down 10 units and right 6 units, horizontally stretched by a factor of $\frac{1}{3}$, reflected in the y-axis, vertically stretched by a factor of 3, and not reflected in the x-axis
- D.** translated up 10 units and left 6 units, horizontally stretched by a factor of $\frac{1}{3}$, not reflected in the y-axis, vertically stretched by a factor of 3, and not reflected in the x-axis

8. Which function represents a vertical translation of 7 units down, a horizontal translation of 8 units right, a horizontal stretch by a factor of $\frac{1}{6}$, no reflection in the y-axis, a vertical stretch by a factor of 6, and no reflection in the x-axis, when compared to the base function $f(x) = \log_9 x$.

- A.** $g(x) = 6 \log_9 \left[\frac{1}{6} (x - 8) \right] - 7$
- B.** $g(x) = \frac{1}{6} \log_9 [6(x - 8)] - 7$
- C.** $g(x) = 6 \log_9 [6(x - 7)] - 8$
- D.** $g(x) = 6 \log_9 [6(x - 8)] - 7$

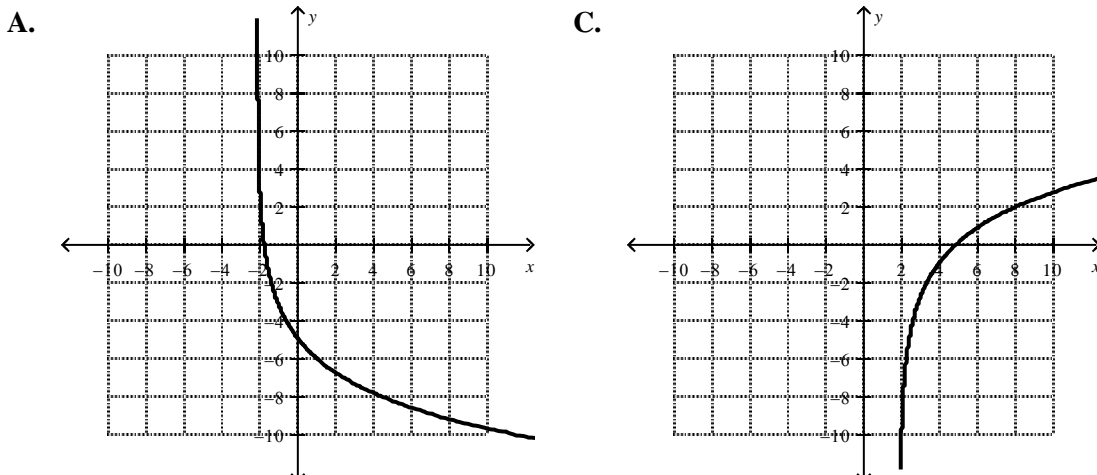
9. Which graph represents the function $y = \log_8(x - 2) - 4$?



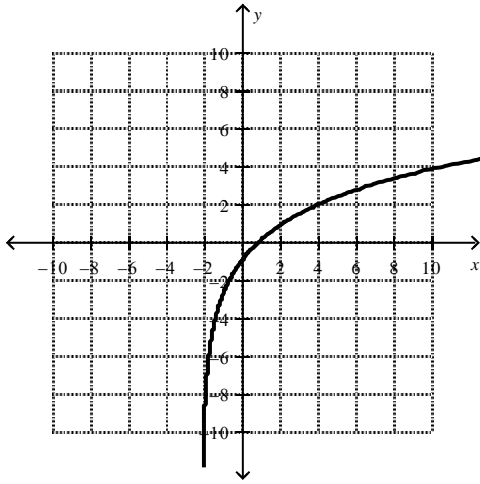
10. What is the equation for the asymptote of the function $f(x) = -\log_7[-5(x + 2)] - 3$?

- A.** $x = 2$
- B.** $x = -3$
- C.** $x = -5$
- D.** $x = -2$

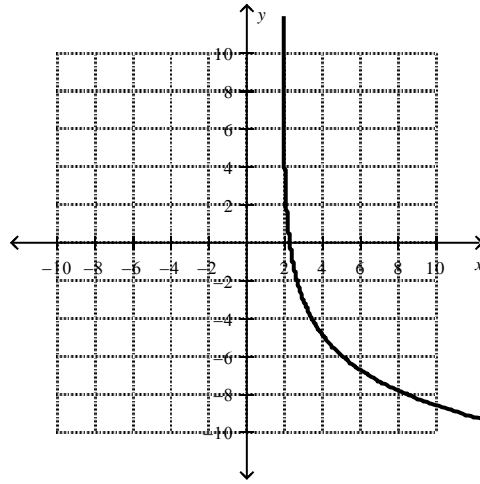
11. Which graph represents the function $y = -3 \log_3[(x - 2)] - 3$?



B.



D.



12. Which of the following is equivalent to the expression $\log_4 s + 7\log_4 v + \log_4 z$? (Workings—3 marks)

A. $\log_4 s v^7 z$

C. $\log_4 7s v z$

B. $7\log_4 s v z$

D. $\log_4 s z + \log_{28} v$

13. If $\log 3 = s$, $\log 5 = v$, and $\log 7 = z$, an algebraic expression in terms of s , v , and z for $\log \frac{5}{441}$ is (Workings 4 marks)

A. $v - 2s + 2z$

C. $v - 2(s - z)$

B. $v - 2(s + z)$

D. $v - 2s + z$

14. Which of the following is equivalent to the expression $\log_4 s w^{10} y$? Workings 3 marks

A. $\log_4 s + 10\log_4 w + \log_4 y$

C. $\log_4 s + \log_4 w + 10\log_4 y$

B. $10\log_4 s - 10\log_4 w + \log_4 y$

D. $10\log_4 s + \log_4 w + \log_4 y$

15. The pH scale is used to measure the acidity or alkalinity of a solution. pH is defined as $\text{pH} = -\log[\text{H}^+]$, where $[\text{H}^+]$ is the concentration of hydronium ions, measured in moles per litre. Determine the pH of a solution with a concentration of $[\text{H}^+] = 4.3 \times 10^{-6}$. Round your answer to two decimal places.

A. 6.00

C. 0.78

B. 5.37

D. 3.52

16. Solve $10^{2x-5} = 7^{x+4}$. Round your answer to two decimal places. (Workings—4 marks)

A. 3.06

C. 2.95

B. 7.26

D. -1.40

17. Solve $\log(3x + 15) = 1 + \log(x + 3)$ to the nearest hundredth. (Workings 4 marks)
- A. -6.43 C. 1.15
 B. 3.46 D. -2.14

Matching

Match the single logarithm in simplest form with the correct equivalent expression.

- | | |
|--|--|
| A. $\log_7 s - \log_7 u + 3\log_7 x$ | D. $8/3\log_7 u - 8\log_7 s + 3\log_7 x$ |
| B. $8\log_7 s - \log_7 u - 3\log_7 x$ | E. $8\log_7 s - 8/3\log_7 u + 3\log_7 x$ |
| C. $8/3\log_7 u - 8\log_7 s - 3\log_7 x$ | F. $8\log_7 s + 8/3\log_7 u + 3\log_7 x$ |

1. $\log_7 \frac{u^{8/3}}{s^8 x^3}$

2. $\log_7 \frac{s x^3}{u}$

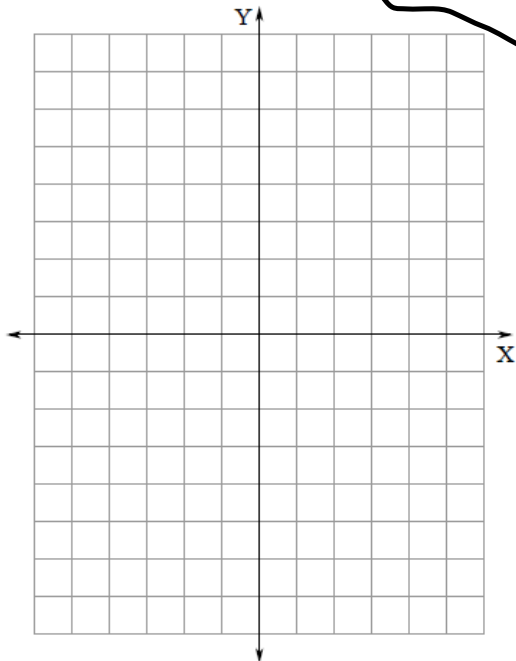
3. $\log_7 \frac{u^{8/3} x^3}{s^8}$

4. $\log_7 \frac{s^8 x^3}{u^{8/3}}$

5. $\log_7 \frac{s^8}{u x^3}$

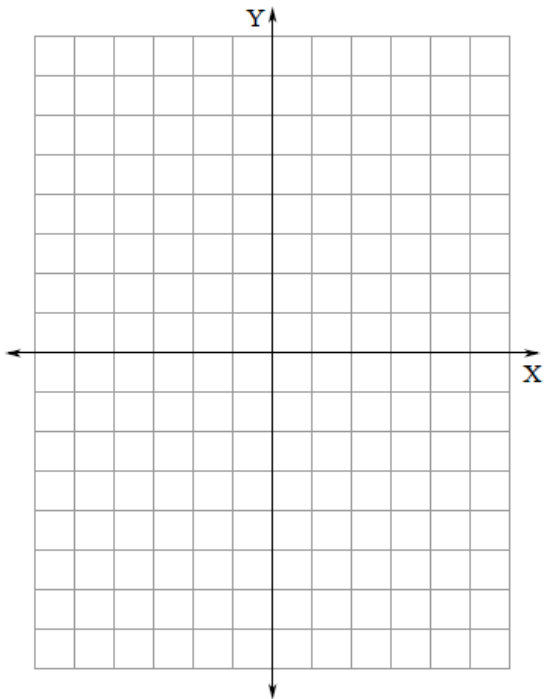
Long Answer

1. Sketch the graph of the function $y = -\log(2x - 6) + 1$. Using a mapping rule, domain and range, x-intercept and y-intercepts. PUT IN PROPER FORM FIRST! 10 marks



Handwritten work:
 $y = -\log(2x - 6) + 1$
 $\sim -\log(2(x - 3)) + 1$
 $\sim -(\log 2 + \log(x - 3)) + 1$
 $\sim -\log 2 - \log(x - 3) + 1$
 $\sim -\log 2 - \log(x - 3) + 1$
 $\sim -\log 2 - \log(x - 3) + 1$
 $\sim -\log 2 - \log(x - 3) + 1$
 $\sim -\log 2 - \log(x - 3) + 1$

2. Graph the function, $y = \log_3(-x + 1) - 2$ using a CORRECT mapping rule. Identify the domain, the range, and the equation of the vertical asymptote. Base Graph New Graph Table 10 marks



3.

A certain type of exponential growth can be described by the equation $N = N_0 10^{kt}$, where N_0 is the initial amount; k is the doubling time, in years; and N is the amount after time, t , in years, has passed. Suppose that the population of a small town doubles every 22 years. Algebraically, how long does it take to triple, to the nearest hundredth of a year?

6 marks

Model:

$$N = N_0 10^{kt}$$

4. Solve the equation A) $6^{3x+1} = 2^{2x-3}$. Leave your answer in exact form as a single logarithm. 7 marks

B) $4 \cdot 3^{x-1} = 10^{-x+2}$

8 marks

5. Solve for x . Be sure to REJECT extraneous solutions.
 $2\log_4(x+4) - \log_4(x+12) = 1$

6 marks

6. Show that $3\log\sqrt{x} + 2\log x - \frac{1}{2}\log x = 3\log x$.

5 marks

7. A 400-g sample of a radioactive substance is placed in a chamber to be tested. After 3 h, 140 g of the sample remains. Determine the half-life of this substance, to the nearest hundredth of an hour.

7 marks

Model:

8. Solve the equation $\log_3 \sqrt[3]{x^2 + 48x} = \frac{2}{3}$.

6 marks

Handwritten work for question 8. The equation $\log_3 \sqrt[3]{x^2 + 48x} = \frac{2}{3}$ is boxed. Below the box is a large scribble of lines. At the bottom, the solutions $x=150$ and $x=2$ are written, with $x=150$ crossed out by a large X.

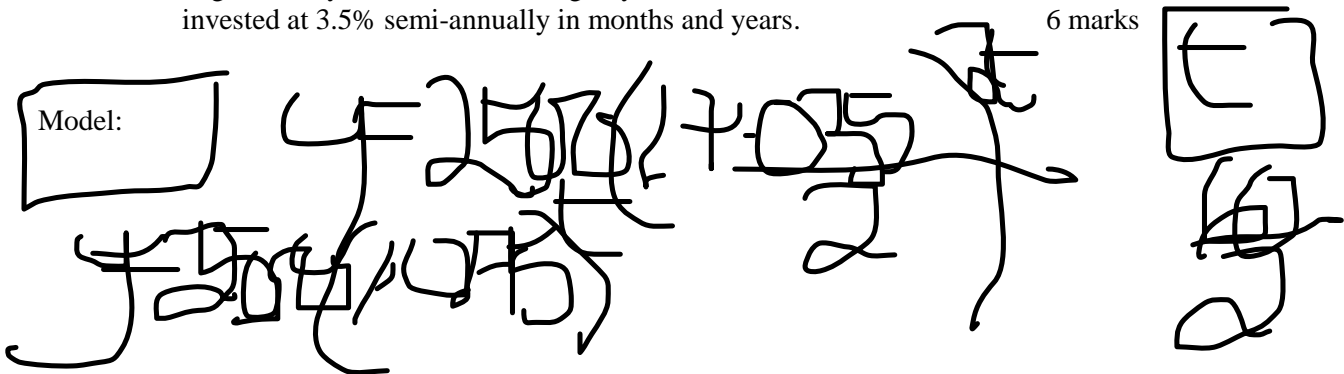
9. The half-life of C14 is roughly 5730 years. If a piece of bone is known to contain initially 1050 mg of C14, determine how long it would take for it to decay to 40 mg to the nearest tenth of a year? To 60% of its initial amount? 2 Qs here! 12 marks

Model:

10 Solve for x: $\log_2(x^2 + 8) = \log_2 x + \log_2 6$ 6 marks

11 Solution A has a PH of 2.2 while solution B has a PH of 4.8. Determine the hydrogen ion concentration for each solution in mol/l and use it to determine how many more times acid is Solution A in comparison to Solution B. 4 marks

12 Algebraically determine how long in years it would take for a \$2500 investment to reach \$6000 if it was invested at 3.5% semi-annually in months and years. 6 marks

Model: 

The handwritten work shows the following steps:

$$6000 = 2500 \left(1 + \frac{0.035}{2}\right)^{2n}$$

$$\frac{6000}{2500} = \left(1 + \frac{0.035}{2}\right)^{2n}$$

$$2.4 = \left(1 + \frac{0.035}{2}\right)^{2n}$$

$$\ln(2.4) = \ln\left(1 + \frac{0.035}{2}\right)^{2n}$$

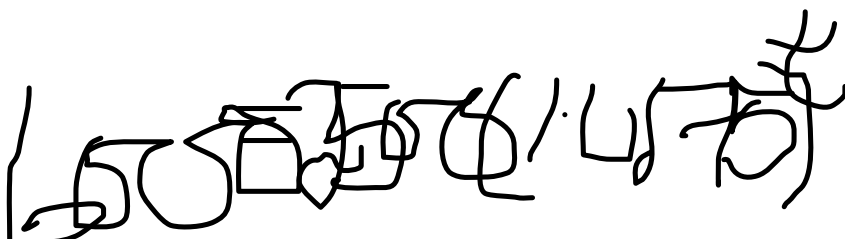
$$\ln(2.4) = 2n \ln\left(1 + \frac{0.035}{2}\right)$$

$$n = \frac{\ln(2.4)}{2 \ln\left(1 + \frac{0.035}{2}\right)}$$

$$n \approx 27.5 \text{ years}$$

13 A radioactive element has 400 g of the substance initially present. After 8 hours only one eighth of the element remains. Algebraically determine the half-life of the element. 6 marks

Model:



The handwritten work shows the following steps:

$$400 \left(\frac{1}{2}\right)^{\frac{t}{T}} = \frac{400}{8}$$

$$\left(\frac{1}{2}\right)^{\frac{t}{T}} = \frac{1}{8}$$

$$\frac{t}{T} = \log_{\frac{1}{2}}\left(\frac{1}{8}\right)$$

$$\frac{8}{T} = \log_{\frac{1}{2}}\left(\frac{1}{8}\right)$$

$$\frac{8}{T} = \frac{\ln\left(\frac{1}{8}\right)}{\ln\left(\frac{1}{2}\right)}$$

$$\frac{8}{T} = \frac{\ln(1/8)}{\ln(1/2)}$$

$$\frac{8}{T} = \frac{-2.079}{-0.693}$$

$$\frac{8}{T} = 3$$

$$T = \frac{8}{3} \approx 2.67 \text{ hours}$$

14 The initial cost of a Ford Escape without taxes right now is \$37,500. It is known the decay at a rate of 20% per year on average. You wish to trade the vehicle in when it is appraised at \$15,000. Determine algebraically when you will trade the Escape in? 6 marks

Model:

15 Questions dealing with the number e? 6 marks

A) Who is the number named after? C) What is its approximation to 6 decimal places?

B) Compute e^5 to 4 decimals. D) Compute $e^{-2} \cdot e \cdot \sqrt[3]{e}$ as an exact value.

14. Simplify as a single logarithm or rational number. 6 marks

A) $\frac{1}{3} \ln 216 + 2 \ln 3 - \ln 6$ B) $\ln 2 + 4 \ln 8 - \frac{1}{2} \ln 64$ C) $\ln 1 - 2 \ln e + 3 \ln 1 + 6 \ln e$

$\ln 1 = 0$

$\ln 1 = 0$

$\ln e = 1$

15 Solve for x leaving your answers as an exact value in terms of e if necessary. 10 marks

A) $\ln(2x+4) = 1$ B) $\ln x + \ln(x-6) = 2 \ln(3\sqrt{2}) + \ln 1$

$\ln 1 = 0$

$\ln 1 = 0$